Taylor Rules and inflation anchoring

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Abstract

The Taylor principle plays a central role in the analysis of inflation anchoring in New Keynesian dynamic general equilibrium models. Ensuring determinacy of equilibria, the Taylor principle attributes to the central bank the power of anchoring inflation. A by-product of this approach is that the Taylor principle sharply reduces inertia in inflation, as it allows to achieve steady state equilibrium with a stroke of a pen. In this paper, we empirically analyse the relationship between inflation inertia and the Taylor rule, focusing on a sample covering the US starting from the 1970s. Two main results of our empirical analysis stand out. First, we find that inflation persistence increased over time across a number of inflation indexes. Second, we find evidence of parameter instability in the monetary policy rule followed by the Federal Reserve Bank across a variety of econometric methods. Building on recent extensions of the standard New Keynesian Dynamic Stochastic General Equilibrium model in which the central bank sets its policy rate on a liquid asset, we propose a parsimonious model that sheds light on our empirical findings. The model, which does not require the Taylor principle for determinacy, reproduces the behaviour of the standard 3-equation New Keynesian Dynamic Stochastic General Equilibrium model (NKSGE) as well as the increased persistence in simulated inflation retrieved in the data.

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1 Introduction

The Global Financial Crisis (GFC) of 2008, and its associated Great Recession, have shattered the consensus on the dominant macroeconomic models, in particular on the Dynamic Stochastic General Equilibrium models (DSGE), which were – and to some extent still are, see Christiano, Eichenbaum, and Trabandt (2018) – a main reference point not only for academic work, but more important, for Central Bank policies. One of the main challenges has been the analysis of monetary policy in this new environment, as the Federal Reserve and other major central banks resorted to policies mainly acting on quantities, with interest rate policies moving to the rear seat. This switch was rationalized as a necessary move due to the interest rate hitting its zero lower bound. However, this was not a simple technical switch in the implementation of monetary policy, as it involved abandoning the so-called Taylor rule, which played a fundamental role in New Keynesian DSGE models. Indeed, those models were useful – with a stable, determinate steady state – as long as the Taylor principle was satisfied: a feedback interest rate rule reacting more than proportionally to inflation was a necessary and sufficient condition for a determinate solution of the models. A determinate solution implies that equilibria are unique and do not depend on arbitrary beliefs by economic actors. Only in such circumstances the model can lead to usable predictions and, more important, can be used for policy analysis. The Taylor principle identifies an active monetary policy, which is consistent with price stability as well as output stability. Therefore, the empirical relevance of the Taylor principle was a key element of the New Keynesian DSGE approach and a consensus had been achieved that active monetary policy was successfully introduced by Volcker, interrupting the previous period of passive monetary policy in the US.

The GFC brought a fundamental problem as at the zero lower bound monetary policy is inherently passive and thus the equilibrium indeterminate.

The implications of indeterminacy are particularly damaging for the dominant macroeconomic models. Indeed, the economy can move along deflationary spirals. One main feature of indeterminacy is that the possibility of infinite equilibrium paths implies that the equilibrium is a function of the beliefs of the economic actors.

Forward guidance turned out to be a crucial instrument to steer expectations during the period with interest rates at the zero lower bound.

However, the awareness of fundamental problems with the standard DSGE models in relation to the determinacy of equilibria is spreading in the profession. These issues are addressed in two ways. One identifies in rational expectations the problem. Accordingly, rational expectations imply that current actions are affected by expectations of variables very far into the future. For example, Garcia-Schmidt and Woodford (2015) assume that, rather than computing perfect foresight equilibria consistent with the announced government policy, economic actors follow an explicit cognitive process in forming expectations on future endogenous variables. Gabaix (2016) builds on a behavioural model in which actors have specific forms of bounded rationality. The result is that current decisions are much less responsive to changes occurring far into the future. The dynamics of an otherwise standard NKDSGE radically change and in particular the model is not subject to indeterminacy at the zero lower bound.

A complementary view, gives more weight to the underlying structure of the NKDSGE models and in particular on its essentially real nature (see for instance Kocherlakota (2016)). For the issue discussed in this paper, namely the Taylor rule, particularly relevant are the recent contributions by Calvo (2016), who puts at center stage in the macroeconomic models issues related to money and liquidity. In contrast to the standard NKDSGE model, the presence of money and liquid assets allows to identify the policy interest rate as an interest rate on liquid assets. Moreover, in such a context, the central bank can act both on interest rates and on quantity of money or liquidity (see also M. B. Canzoneri and B. T. Diba (2005) and M. Canzoneri et al. (2008a,b)). Interestingly, the model loses the knife-edge property on the coefficient of the Taylor rule typical of NKDSGE models. Indeed, when
the central bank sets a constant total amount of liquidity, determinacy is ensured irrespective of the coefficient on inflation in the Taylor rule. Hence a Central Bank committed to anchoring actual and expected inflation does not need to comply with the Taylor principle.

These considerations have relevant implications for empirical analysis. Parameter instability in an estimation of a Taylor rule equation is consistent with inflation under control. It reflects structural change and emergence of uncertainty episodes.

We contribute to this discussion in three ways. First, we document that across a variety of indexes, inflation persistence is increasing, making inflation rates more sticky than past decades. We obtain this insight from a purely statistical standpoint, without economic assumptions or restrictions.

Second, we connect this to fundamental instability in the monetary policy conduct, as embodied by the Taylor Rule. In particular, we estimate several specifications of the rule with a number of methods, from simple OLS to Markov Switching models. We find a significant amount of instability in the estimates, varying over methods, specifications, and sample cuts – even when the Taylor principle was supposedly respected. In line with Murray, Nikolsko-Rzhevskyy, and Papell (2015) and Sullivan (2016), we also find that the simple, exogenous, distinction between a pre- and post-Volcker regime does not seem to be robust. We find evidence of the presence of at least two regimes associated with sizeable differences in key parameter values of the Taylor rule. A relevant finding is that parameter values crucially depend on the number of regimes considered. Nevertheless, some regularities emerge: financial conditions seem to matter when the Fed conduces its policy, in particular when we consider a measure of liquidity in our estimates. To proxy for financial liquidity we consider spreads between safe and moderately risky assets, namely Treasury bills, shares indexes, and corporate debt.

Third, we consider an off-the-shelf, stripped-off NKDSGE model and augment it with stylised liquidity features. Such slight departure (as opposed to more complex mechanisms like B. Diba and Loisel (2017)) is sufficient to obtain interesting results: the Taylor Principle does not constitute a requirement for a determinate, stable solution when the Central Bank targets the interest rate on a liquid bond to connect it with inflation. We also study the effects of productivity and monetary policy shocks in our setting and compare the effectiveness of two monetary regimes, either a passive ($\gamma \leq 1$) or active ($\gamma > 1$) Central Bank. We find small difference across the two regimes, mostly comparable in magnitude: this holds true even comparing our liquidity model against the baseline NKDSGE. Our model successfully reproduces the standard results of basic NKDSGE models, both in signs, profiles, and magnitude of the impulse response functions. Moreover, we also study the consequences of a liquidity dry-up under the two regimes, showing how, in such event, an active monetary policy might help tame the effects of this shock and keep the consequences in the financial market.

Moreover, in connection to our findings on inflation persistence, we study the properties of the simulated inflation series generated in our model and the baseline NKDSGE. We show that an accommodative monetary policy generates significantly more persistent inflation rates. This corroborates the intuition that originally motivates our study, the interplay of monetary policy regimes and inflation dynamics properties.

The paper is structured as follows: Section (2) explores the recent changes in the dynamic properties of several inflation measures, Section (3) presents an overview of the macro- and micro-economic data we collected in our database, Section (4) offers an empirical study of the Taylor rule estimates in different specifications and obtained with different methods, Section (5) presents a model that systematises the findings of our empirical work, Section (6) concludes.

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1This suggests the potential difficulty in identifying the parameter and it is consistent with findings by Canova (2007) and Canova and Sala (2009), who argue that in Bayesian approach the estimated parameter depends critically on the prior distribution that is assumed.
2 Inflation dynamics

In the last decade, inflation was – and still is – part of a lively conversation on monetary policy, especially in connection with the consequences of the new, unconventional tools Central Banks adopted to curb the GFC consequences (Taylor, 2014). In spite of the unprecedented monetary expansion by US and European Central Banks, inflation remained stubbornly low for a long period. This phenomenon has raised questions on the applicability of the dominant NKDSGE model for explaining inflation dynamics. In this light, it is thus useful to review the behaviour of inflation and its dynamic properties over the post WWII period in the US economy.

Contrary to other studies like Cogley, Primiceri, and Sargent (2008), we exclusively focus on the behaviour of inflation as a stochastic process generating the time series that we observe. We postpone the analysis of inflation in connection to the Taylor rule pursued by the Central Bank to Section (4).

A first issue to tackle is the specific index of inflation to analyse, as there exists multiple measures of inflation. In consideration of this multiplicity, we employ and compare a variety of time series. Moreover, a key issue is related to timing: real-time, now-cast series are those available to policymakers at the time of their decisions, while revised data are more reliable as they result from more information, but they become available too late for timely use in policy.

We consider three classes of inflation indexes. The Consumer Price Index (CPI), the Personal Consumption Expenditure index (PCE), and finally the Gross Domestic Product Deflator. These three indexes are measured on different baskets of goods, hence discrepancies and deviations are due to the distinct subset of goods and services each index tracks. Specifically, the CPI mainly relates to consumers purchases, the PCE relates to business sales while the GDP deflator is measured on the goods and services produced within the US territory, abstracting from import prices. CPI and PCE also differs in the weights for each good and are available as “headline” and “core”, with the latter excluding volatile items like food and energy.

In our study we employ revised CPI (headline and core), revised PCE (headline and core), revised GDP deflator, CPI, PCE and deflator nowcasts, and finally one-period-ahead deflator forecasts. While the most useful information for our study comes from the historical series, we also include nowcasts and forecasts to assess how inflation expectations are formed. Most series start before 1960, with the exception of CPI and PCE nowcasts, which start in 1979 and 1986, respectively.

In defining the methodologies to use for studying inflation dynamics, we partially follow (and further extend) the instructive collection of techniques presented in Pivetta and Reis (2007) and Fuhrer (2011), with which this work partly overlaps.

The time series used are provided by two regional Federal Banks: that of St. Louis and that of Philadelphia. The latter, in particular, published the Greenbook dataset and the Survey of Professional Forecasters, which we used in our study. St. Louis Fed provides a wealth of revised time series on the US economy.

All series offer quarterly observation of annualized growth rates. Figs. (14), (15),(17) plot the time series we collect: forecasts, nowcasts, and historical data, respectively.

2.1 AR (1) process

The most immediate tool available to investigate the dynamic properties of a time series is a simple autoregressive process with only one lag. This naive analysis is useful as it allows for simple unit root tests. Hence, the straightforward model estimated is

\[ \pi_t = \mu + \rho_1 \pi_{t-1} + \epsilon_t \]  

(1)

The results of a simple OLS regression are summarised in Table (1). At this point it is not possible
to make any claims about the persistence of inflation, as these estimates cannot be contrasted with any threshold for stickiness or lack thereof. Nevertheless, based on the estimates, we can say that nowcasts and forecasts are generally less persistent than historical series. These latter indicate $\rho$ estimates all well above 0.9.

Additionally, Table (1) contains results of the augmented Dickey-Fuller test for the presence of unit root in each series. In this test, the null assumes presence of a unit root. Only the CPI nowcast does not present evidence of unit root according to the ADF test.

### Table 1: AR (1) estimates on the full available sample.

<table>
<thead>
<tr>
<th></th>
<th>CPI$_{t:t}$</th>
<th>PCE$_{t:t}$</th>
<th>DEFL$_{t:t}$</th>
<th>DEFL$_{t:t+1:t}$</th>
<th>CPI$_{head}$</th>
<th>CPI$_{core}$</th>
<th>DEFL</th>
<th>PCE$_{head}$</th>
<th>PCE$_{core}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>1.3644***</td>
<td>.7838***</td>
<td>.5362**</td>
<td>.2644*</td>
<td>.1926*</td>
<td>.0895</td>
<td></td>
<td>.1074+</td>
<td>.5221**</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>(.9636)</td>
<td>(.0677)</td>
<td>(.0379)</td>
<td>(.0282)</td>
<td>(.0199)</td>
<td>(.0144)</td>
<td></td>
<td>(.0156)</td>
<td>(.0256)</td>
</tr>
<tr>
<td>obs.</td>
<td>132</td>
<td>107</td>
<td>183</td>
<td>183</td>
<td>281</td>
<td>241</td>
<td></td>
<td>281</td>
<td>231</td>
</tr>
<tr>
<td>adj. $R^2$</td>
<td>.3808</td>
<td>.5084</td>
<td>.738</td>
<td>.856</td>
<td>.8889</td>
<td>.9499</td>
<td></td>
<td>.9312</td>
<td>.8478</td>
</tr>
<tr>
<td>BIC</td>
<td>612</td>
<td>274</td>
<td>621</td>
<td>486</td>
<td>794</td>
<td>431</td>
<td></td>
<td>531</td>
<td>720</td>
</tr>
<tr>
<td>ADF sc.</td>
<td>-2.8982</td>
<td>-1.4154</td>
<td>-1.1517</td>
<td>-1.2075</td>
<td>-1.1365</td>
<td>-0.9835</td>
<td></td>
<td>-0.8941</td>
<td>-0.536</td>
</tr>
<tr>
<td>null rej.</td>
<td>.99%</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td></td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>obs.</td>
<td>132</td>
<td>107</td>
<td>183</td>
<td>183</td>
<td>281</td>
<td>241</td>
<td></td>
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<td>.8889</td>
<td>.9499</td>
<td></td>
<td>.9312</td>
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<td>531</td>
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<td>-0.9835</td>
<td></td>
<td>-0.8941</td>
<td>-0.536</td>
</tr>
<tr>
<td>null rej.</td>
<td>.99%</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td></td>
<td>no</td>
<td>no</td>
</tr>
</tbody>
</table>

### 2.2 AR (5) process

Extending the lags included in the regression sheds light on more complex behaviour for inflation. Next step consists in allowing for the estimation of an AR (5) process: time $t$'s inflation might depend on the value it had fifteen months earlier. This step is a pathway towards optimal lag estimates and eventually rolling window OLS. This last approach, in particular, will provide insights on how $\rho_1$ changes over time for the time series considered.

### Table 2: AR (5) estimates on the full available sample.

<table>
<thead>
<tr>
<th></th>
<th>CPI$_{t:t}$</th>
<th>PCE$_{t:t}$</th>
<th>DEFL$_{t:t}$</th>
<th>DEFL$_{t:t+1:t}$</th>
<th>CPI$_{head}$</th>
<th>CPI$_{core}$</th>
<th>DEFL</th>
<th>PCE$_{head}$</th>
<th>PCE$_{core}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>1.1589***</td>
<td>.2156</td>
<td>.2569</td>
<td>.1698</td>
<td>.1963*</td>
<td>.1015+</td>
<td></td>
<td>.0884+</td>
<td>.4263*</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>(.3404)</td>
<td>(.1963)</td>
<td>(.1676)</td>
<td>(.1157)</td>
<td>(.0795)</td>
<td>(.0592)</td>
<td></td>
<td>(.0474)</td>
<td>(.1908)</td>
</tr>
<tr>
<td>obs.</td>
<td>128</td>
<td>103</td>
<td>179</td>
<td>179</td>
<td>277</td>
<td>237</td>
<td></td>
<td>277</td>
<td>229</td>
</tr>
<tr>
<td>adj. $R^2$</td>
<td>.3073</td>
<td>.6509</td>
<td>.7915</td>
<td>.8833</td>
<td>.9255</td>
<td>.9626</td>
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<tr>
<td>BIC</td>
<td>567</td>
<td>244</td>
<td>587</td>
<td>458</td>
<td>689</td>
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<tr>
<td>ADF sc.</td>
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<td>-1.157</td>
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<td></td>
<td>-0.536</td>
<td>-0.7133</td>
</tr>
<tr>
<td>null rej.</td>
<td>.99%</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td></td>
<td>no</td>
<td>no</td>
</tr>
</tbody>
</table>

Table (2) presents the estimates for the coefficient on the first lag, when the process is an AR (5). The same overall behaviour highlighted above emerges in this table: nowcasts and forecasts are less persistent – depend less on past realisations – than revised series. For the revised series in some cases coefficients are greater than one. Therefore, we measure persistence as distance from the limit case of unit root, when $\rho_1 = 1$. 


2.3 Rolling window $AR (k^*)$

To pull the most information from this methodology, we sequentially increase the lags number $k$ until we find an optimal length for each series. Once the optimal $k$ is found, then, we could compare every $k_i$ with the other to tell which series is the most persistent. Whilst this is an interesting exercise, it would give us little information on the evolving dynamic properties of inflation. To provide useful insight with respect to varying persistence, we estimate an $AR (k^*)$ with a rolling window, where $k^*$ is the optimal number of lags, and plot the estimated $\rho_1$. The resulting sequence of estimates provide information on the changes of persistence in one particular time series.

\[
\pi_t = \mu + \sum_{i=1}^{k^*} \rho_i \pi_{t-i} + \epsilon_t \quad k^* = \arg\min BIC (AR (k^*))
\] (2)

As of the width of the window, it influences the smoothness of the change. A large window, although providing precise estimates, takes much more observations to detect a change in the estimated parameter. On the other hand, a smaller window signals starkly the change, but with a greater level of uncertainty around the estimate. Therefore, the trade-offs must balance precision, sensitivity, and also data availability. Some of our series are relatively short, an excessively large window would leave us with few estimates.

Keeping these considerations in mind, we set our window length to 56 observations, as done in Pivetta and Reis (2007) and Fuhrer (2011), summing to 14 years. Shorter lengths are presented in the Appendix (B) as robustness checks.

Fig. (1) present the results of the rolling window OLS. We focus on revised data mostly and present only one forecast series – this latter will be employed in the subsequent sections. The GDP deflator one-period-ahead forecast, on the bottom right pane, reports stable and reasonably low persistence in the first lag estimate over the years. The implications are that forecasters have maybe improved little their techniques for inflation, as no breakthrough technology has raised the reliability of past forecasts. As this results is remarkably similar to the now-casts series, our comments apply to those series as well.

Historical series deserve more attention. Starting from the “outliers”, headline PCE and core CPI (mid left and top left panes) reports that $\rho_1$ is moving away from the neighbourhood of 1, meaning that inflation is becoming less sticky according to these measures. This result is less stark when focusing on the post GFC date, when the estimate do not move as much as in the past years.

The remaining panels of Fig. (1) picture an overall increase in the relevance of the first lag, an increase in the persistence of inflation over several measures. This seemingly corresponds to a trend that started in the mid-Eighties – as it is clear from the left top and mid panels of headline CPI and the GDP deflator. A secular trend might also be observed in the core PCE (bottom left pane). This series trends down until the onset of the GFC, then bounces back towards 1.

In summary, the change in inflation dynamics seems to take place in the mid-eighties, and accelerates after the GFC. Therefore, monetary policy is a plausible candidate for the change of dynamic properties of inflation.

In the following Sections we study the interplay between inflation and other macroeconomic aggregates to retrieve the effects – stabilising or destabilising – of monetary policy. For such a goal, we focus on the Taylor rule typically imputed to the Federal Reserve Bank, and investigate its stability over the post-WWII decades.
Figure 1: AR $(k^{*})$ estimates of $\rho_1$. Solid black line is the point estimate on $\rho_1$, red bands mark 2SE area. In blue, a polynomial LOESS fit conveys the overall trend from local observations.
3 Data

In order to conduct our empirical analyses, we build a database of the most relevant time series on the US economy aggregates. It includes historical and real time data at the macroeconomic level, as well as statistics from specific microeconomic data. At a broad level, we collect data on inflation, interest rates, real output slack, monetary aggregates, government debt and deficit, financial market indicators, and finally (measures of) expectations of these variables. For each one of these aggregates, we collect a set of more specific measures that differ in the exact definition or computation: the clearest examples are the GDP deflator, the CPI, and the CPE for inflation, or the capacity utilization, lay-off rate, unemployment, and Fed’s own calculations for the output gap.

Concerning the micro data, we exploit the information present in the Greenbook dataset: it contains the Survey of the Professional Forecasters, that provides expectations on the current and future perception of the economy. The financial information we employ relies on two measures: the quarterly returns of the S&P 500 index and the weighted average return of BAA corporate bonds. These series are then used as proxies for the liquidity in the economy.

The vast majority of the series are retrieved from the websites of the Federal Reserves of St. Louis and Philadelphia: a complete list is provided in the Appendix, Table (13).\footnote{The resulting dataset, as well as the code to compile and maintain it, are available at this Git repository. The code itself might undergo significant improvements and variations over time. Since at each run the latest data are fetched, resulting estimates might vary.}

For some of the data we use, we performed operations like filtering or extraction, to isolate precise information from raw data. While most cases are straightforward, like taking annualized growth rates, others deserve a brief explanation, which we summarise below.

Output gap  Beginning with the output gap measures, we included three different manners. First, its direct estimate in real time: for each available date \( t \), we regress the time series against a quadratic time trend and finally take the residual of the latest available data point, \( \epsilon_t \), as output gap observation for date \( t \). This extrapolation uses data from the Greenbook database on the real time estimates on the GDP level and implements the methodology mentioned in Murray, Nikolsko-Rzhevskyy, and Papell (2015). We label the resulting time series as \textit{real time output gap}.

Second, we compute the percentage difference between installed capacity and actual GDP, both provided by St. Louis Fed. We call this series \textit{ex post output gap} since it relies on historical, revised data, not necessarily those available to policy makers at the time of their contingent decisions.

Third, we also use the lay-off rate on total employment, a measure put forward by Berger et al. (2016).


Inflation and expectations  Concerning the measures of inflation, we include among revised time series the indexes of GDP deflator, the Consumer Price Index (CPI), and the Personal Consumption Expenditure (PCE). For the last two, we also include their versions excluding food and energy prices, dubbed Core CPI and PCE ELF.

The Greenbook database from the Philadelphia Fed provides information on last, current and future values for three of the aforementioned indexes, namely CPI, Core CPI, and GDP deflator. In particular, expectations – or more thoroughly, forecasts – are available up to eight quarters ahead from...
time $t$. These expectations are part of the information set of the policy-maker at the time of its decision and thus represent a more reliable tool to gauge the policy function in place.

**Fiscal and monetary data** We include in our database information about the fiscal position of the economy as well as the classic monetary aggregates tuned by the Central Bank. The main purpose is to flexibly test for some fashion of the Fiscal Theory of Price Level (FTPL) (see for example Cochrane (2011) and Leeper (2010)) and more classical monetarist theories. To this purpose, we include base money, M1, and M2. For the FTPL we include measures of government deficit over GDP, total public debt, public debt held by the Federal Reserve (all these in growth rates, levels, and shares when possible, too).

**Liquidity proxies: financial indexes** After the financial markets collapse that triggered the Global Financial Crisis in 2008, liquidity in the economy gained momentum as research topic alongside with safe assets and thus risk, especially following the massive injections carried out by the Federal Reserve, see among others Caballero, Farhi, and Gourinchas (2016, 2017), M. B. Canzoneri and B. T. Diba (2005), M. Canzoneri et al. (2008a,b), Del Negro et al. (2017), and Hall and Reis (2016).

Our idea hinges on using the condition of the financial markets to parse out information on financial liquidity. Financial market prices embody plenty of different information, so the risk of picking up the wrong signal or incur in plain endogeneity is high. Considering these threats we compute our indicators as premia over safe assets of comparable maturity, which are subject to “fire-purchases” in times of uncertainty or economic turmoil.

The simple intuition goes as follows: on the edge of a recession or slowdown, publicly traded assets are fully liquid\(^3\) and smoothly traded; when uncertainty kicks in or expectations turn pessimistic, these assets become second choice to more reliable, safer assets. Therefore, the spread between the former and the latter factors in the variation in the liquidity of the the economy.

We include these spreads in the decision rule of the Central Bank in order to test whether policy makers are also attentive to the liquidity in the economy and act to accordingly.

4 The Taylor rule through the decades, the specifications, and the methods

In this section we propose a set of estimates of the decision rule followed by the monetary authority. We estimate these rules with methods that allow for parameter instability. To begin with, we exogenously split the sample in three sub-samples and compare the parameters. Then, we estimate the rule on the full sample and investigate possible structural breaks. Third, we let the sub-sampling be somewhat endogenous with a Markov Switching estimation for two possible states.

All the methods above are tested over a variety of specifications of the Taylor rule, so to assess the robustness of the traditional specification compared to the alternatives. Our interest lies particularly in the parameters stability over different methods and specifications.

Throughout this Section, we will estimate equation (3), which encompasses all the information detailed above. In this specification $r$ is the effective federal fund rate, $\pi_{t+h}$ is $h$-period ahead inflation expectation (mapped to forecasts, up to 8-quarter), $\hat{y}$ is output gap in percentage deviation, and $x$ is a vector collecting any additional variables used in the study as detailed in Section (3).

\(^3\)Ask/bid gap is close to zero.
In equation (3), we assume that the Central Bank smooths its policy decision putting a weight \( \rho < 1 \) on past interest rate level. Therefore, we need to recover estimates and confidence intervals from the estimated \( \rho \). Moreover, we allow for inflation targeting including an intercept \( \mu \).

We will carefully focus on two parameters, \( \beta \) and \( \gamma \). The sign and the magnitude of the former will tell how relevant other factors are for the Central Bank; on the other hand, \( \gamma \) will shed light on the robustness of the Taylor Principle. Established consensus points to a value close to \( \gamma = .8 \) following the onset of the Great Moderation and the inflation conquest carried out by Volcker.

Estimating the Taylor rule with a generous variety of data yields results that are prima facie consistent with the consensus view according to which the Federal Reserve Bank (over-) reacted to expected inflation in recent decades.

In the literature there is a broad consensus on the empirical validity of the Taylor rule at least since the chairmanship of Volcker. Volcker, the consensus goes, induced a switch in policy from a regime of indeterminacy (accommodation policy) to one of determinacy. One of the first attempts to verify such break in policy is Clarida, Gali, and Gertler (2000), who exogenously divide their sample in two periods and obtain estimates for each one. They find that the FED was following a passive monetary policy during the first part of the sample, whereas an active policy emerged after Volcker chairmanship, resulting in a miraculously stable inflation path over the whole post-Volcker period.

By contrast, Boivin (2006) uses a different approach to let the data speak, estimating a Taylor rule with drifting parameters over the sample. He finds that inflation response was weak in the second half of the 1970s, but strong in the rest of the sample. The response to real activity, measured as the deviation of the unemployment from its natural rate, decreased significantly and permanently after the 1970s. Starting from the mid-1980s he finds that stability in the policy parameters increased. These changes happened in an unsynchronised way, with the most important changes occurring between 1980 and 1982, the transition years of Volcker chairmanship.

Another approach to the endogenisation of policy changes is found in Murray, Nikolsko-Rzhevskyy, and Papell (2015), where Hamilton’s algorithm for Markov processes estimation is applied to monetary policy rules. As is the case of the other contributions, monetary policy is not stable and its parameters change over time. Murray, Nikolsko-Rzhevskyy, and Papell (2015) explore the two-state case, finding two periods of undetermined policy, precisely 1973:1975 and 1979:1985, again in coincidence of the transition period before inflation was “conquered”. The point estimates of these three contributions are summarized in the tables below.

<table>
<thead>
<tr>
<th>Table 3: Clarida, Gali, and Gertler (2000)</th>
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</thead>
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<td>Exogenous break</td>
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<tr>
<td>Post-Volcker</td>
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<tr>
<td>( \mu )</td>
</tr>
<tr>
<td>( \gamma )</td>
</tr>
<tr>
<td>( \omega )</td>
</tr>
<tr>
<td>( \rho )</td>
</tr>
<tr>
<td>4.24</td>
</tr>
<tr>
<td>-.27</td>
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<tr>
<td>1.15</td>
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<table>
<thead>
<tr>
<th>Table 4: Murray, Nikolsko-Rzhevskyy, and Papell (2015)</th>
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<td>Markov State</td>
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<tr>
<td>( S_2 )</td>
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<tr>
<td>( \mu )</td>
</tr>
<tr>
<td>( \gamma )</td>
</tr>
<tr>
<td>( \omega )</td>
</tr>
<tr>
<td>( \rho )</td>
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<td>.85</td>
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<td>.58</td>
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</table>

On the basis of the results below, we propose that the change in the dynamic properties of inflation is connected to the instability of the response of the Central Bank through the Taylor Rule.

Before exposing the results, we precise the eight specifications we estimate throughout this Section and briefly motivate their utilization.
Spec. I \( r_t = (1 - \rho) [\mu + (1 + \gamma) E_t \pi_{t+1} + \omega \hat{y}_t] + \rho r_{t-1} + \epsilon_t \): the standard specification as in Taylor (1993) and many other works. We employ one period ahead forecasts of GDP deflator as expected inflation and real-time gap for output slack.

Spec. II \( r_t = (1 - \rho) [\mu + (1 + \gamma) E_t \pi_{t+1} + \omega \hat{y}_t] + \rho r_{t-1} + \epsilon_t \): we replace the real-time output gap with the lay-offs over total employment, to account for one leg of the Federal Reserve’s mandate.

Spec. III \( r_t = (1 - \rho) [\mu + (1 + \gamma) E_t \pi_{t+1} + \omega \hat{y}_t + \beta \text{BAA}] + \rho r_{t-1} + \epsilon_t \): this is the first specification accounting for financial stress in the economy. To this purpose, we add to Spec. I the first proxy for liquidity condition in the economy, as captured by the spread between BAA corporate bonds and 10 years Treasury bonds.

Spec. IV \( r_t = (1 - \rho) [\mu + (1 + \gamma) E_t \pi_{t+1} + \omega \hat{y}_t + \beta \text{S&P 500}] + \rho r_{t-1} + \epsilon_t \): we test a second proxy for liquidity with this specification. We exploit quarterly returns on the stock market to obtain a spread with 3-month Treasury Bills.

Spec. V \( r_t = (1 - \rho) [\mu + (1 + \gamma) E_t \pi_{t+1} + \omega \hat{y}_t + \beta \text{S&P 500}] + \rho r_{t-1} + \epsilon_t \): in this specification we blend Spec. II and IV.

Spec. VI \( r_t = (1 - \rho) [\mu + (1 + \gamma) E_t \pi_{t+1} + \omega \hat{y}_t + \beta \text{BAA}] + \rho r_{t-1} + \epsilon_t \): in this specification we blend Spec. II and III.

Spec. VII \( r_t = (1 - \rho) [\mu + (1 + \gamma) E_t \pi_{t+1}^{\text{SPF}} + \omega \hat{y}_t] + \rho r_{t-1} + \epsilon_t \): this specification replaces the information on inflation with the mean forecast \( E_t \pi_{t+1}^{\text{SPF}} \) from the Survey of Professional Forecasters that the Federal Reserve system polls for expectations. This specification reflects closely the market expectation on inflation.

Spec. VIII \( r_t = (1 - \rho) [\mu + (1 + \gamma) E_t \pi_{t+1}^{\text{SPF}} + \omega \hat{y}_t + IQR(\pi_t^{\text{SPF}})] + \rho r_{t-1} + \epsilon_t \): this specification includes a proxy for uncertainty in the economy, specifically regarding inflation. This proxy is the interquartile range in the cross-section of the SPF at date \( t \). The higher the dispersion, the higher the uncertainty in the economy about the inflation process. This metric reflects the intuitions and the findings of Section (2).

We decided to select and present these specifications because other combinations, although conceptually appealing, do not necessarily add interesting insights. We briefly present them in the Appendix.

**OLS on the full sample** The first step is to estimate several specifications on the full sample, ignoring the possibility of structural breaks or fluctuations in the parameters. The sample ends in 2018Q2, with starting date varying according to the variables considered: most of the specifications cover more than 180 observations, only three have less than 150 observations. Table (5) summarises estimates for several models.

These results are interesting in a number of aspects. First, the sample encompasses a variety of regimes: from the pre-Volcker era to the ZLB period, with the Great Moderation data dwarfing other regimes. Hence it blends together different rules and behaviours with diverse weights.

Second, according to the specification, parameters estimates mark large variations, especially in traditional regressors. This parameter instability might arise from the particular features of the sample’s beginning and end. On the other hand, these chunks of data add the necessary variation to thoroughly test the Central Bank behaviour under different situations.

Third, specifications III to VI dispute the consensus on the Taylor Principle. In fact, the inclusion of liquidity proxies significantly lowers the weight on inflation expectations, down to levels violating the Taylor Principle. Across these specifications, though, there is a remarkable stability in the lay-off rate and real time output gap, although significance varies. Conditional on the type of real slack, expected inflation weights are also somewhat stable, generating the most interesting results when considering our real time output gap instead of lay-offs.
Table 5: OLS estimates in the full sample, up to 2018 Q2

<table>
<thead>
<tr>
<th>Spec.</th>
<th>$\mu$</th>
<th>$E_t$ (GDP defl.)</th>
<th>$\delta_t$</th>
<th>Lay-off</th>
<th>BAA spr.</th>
<th>SP spr.</th>
<th>$E_t$ (CPI SPFt+1)</th>
<th>$IQR_t$ (CPI SPFt+1)</th>
<th>FFRt-1</th>
<th>Obs.</th>
<th>$R^2$</th>
<th>BIC</th>
</tr>
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<tbody>
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<td>I</td>
<td>1.334</td>
<td>1.5803***</td>
<td>0.496**</td>
<td>0.582**</td>
<td>-2.5665***</td>
<td>0.7781***</td>
<td>304.75**</td>
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<td>619.05</td>
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<tr>
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<td>3.514**</td>
<td>2.4490***</td>
<td>2.659**</td>
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<td>3.485</td>
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<td>3.485</td>
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<td>519.05</td>
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<td>2.5665***</td>
<td>-3.014</td>
<td>-3.536**</td>
<td>184</td>
<td>108</td>
<td>156.06</td>
<td>9711</td>
<td>156.06</td>
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<tr>
<td>IV</td>
<td>0.303</td>
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<td>-4.318***</td>
<td>688.82***</td>
<td>688.82***</td>
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<td>VI</td>
<td>13.89**</td>
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<td>1.209**</td>
<td>-1.501</td>
<td>-3.028**</td>
<td>302.66**</td>
<td>302.66**</td>
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<td>152.95</td>
<td>9719</td>
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<td>9538</td>
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</table>

Significance codes: *p<0.1; **p<0.05; ***p<0.01; SE in parentheses.

In all cases, liquidity proxies matter in the policy decisions of the Federal Reserve. This result is even more surprising when considering that the BAA spread series starts in 1986Q1, at the end of Volcker’s mandate.

To further study the parameters instability presented in Table (5) we split the sample in three sub-samples, upon which we cast the assumption of different regimes.

**Exogenous breaks** We split the sample to obtain three phases:
1. pre-Volcker regime [-:1979Q2],
2. the Great Moderation [1980Q1-2007Q2],
3. and finally the Global Financial Crisis [2008Q1-:]

Historically, the period covered by sub-sample (i), has seen high inflation and federal fund rate as well as ample cyclical fluctuations. According to Clarida, Gali, and Gertler (2000), among others, the Fed carried out an accommodative monetary policy along those years, following inflation instead of aggressively responding to its expectations. Hence, we expect to see values close to those presented in Table (3).

Unfortunately, data availability limits the estimation of some of our specifications: Specs III, VI, and VII cannot be estimated for the first period.

The second chunk of data covers the inflation conquest and the miraculous steady and sustained growth that followed, with mild recessions and inflation in check. Supposedly, this conditions were brought about by a Central Bank eventually fighting back inflation aggressively.

The third period starts right before the Global Financial Crisis. Data are still scarce: to date, we have about 10 years of quarterly data with hardly enough variation, mainly because of the FFR hitting the zero lower bound and hovering in its neighbourhood. Therefore, the estimates here shall be considered cum grano salis.

Skimming through Table (6) it is interesting to compare the regimes in place. Although the heterogeneity in available observations restricts significantly the econometric validity of such exercise, a number of regularities emerges.

We start from the first sub sample, prior to the Great Moderation. Contrary to the established consensus, we find that the Fed was not accommodating inflation if we consider a regular Taylor Rule (Spec. I and II) with $\gamma > 0$. On the other hand, if we assume that it was not strictly following such rule

*Appendix (C.2) offers the residuals plot for the full sample regression. Eyeballing these plots provides sufficient motives to carry out additional analyses on model instability.*
Table 6: Exogenous splits: three samples

<table>
<thead>
<tr>
<th>Spec.</th>
<th>Sample</th>
<th>$\mu$</th>
<th>IT ($\Delta$GDP dell $\Delta$1,2)</th>
<th>Real time $\gamma$</th>
<th>Lay-off</th>
<th>BAA spr.</th>
<th>SP spr.</th>
<th>$E_{t}^{(CPIT_{T}^{u})}$</th>
<th>$IQR_{t}^{(CPIT_{T}^{u})}$</th>
<th>FF $R_{t-1}$</th>
<th>Obs.</th>
<th>$R^{2}$</th>
<th>BIC</th>
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<td>(1.9053)</td>
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<td>(1.1938)</td>
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</table>

Significance codes: *p<0.1; **p<0.05; ***p<0.01; SE in parentheses. (i) covers from the earliest available observation to 1979Q2; (ii) covers from 1980Q1 to 2007Q2; (iii) goes from 2008Q1 to the latest observation available, in this version it is 2012Q2, as some data are published with a five years lag.

and include financial conditions (Spec. IV), then our estimates are precisely in line with those on the full sample (Tables (5), (3), (4)).

Looking at Spec. III to VI and comparing the estimates over the sample cuts, we find wide confirmations of the instability previously reported, although constrained by limited sample size.

Among the different specifications we considered, those involving some measure of liquidity do perform relatively well, especially over the longer sample 1980–2007. Financial liquidity likely enters the information set of the Federal Reserve policy decisions: when liquidity dries up because of financial or real turmoil (and hence spreads increase) the monetary authority puts in place accommodative policies by decreasing the reference interest rate. This applies both on short term assets (SP spread) and longer maturities (BAA bonds), as the monetary impulse whips through the yield curve.

Estimates on the latest sample deserve comment and explanation. Over its few observations, the key policy rate barely moves, with other variables displaying more variability. These facts explain why in all regression the most significant variable is the lagged interest rate, with all others rarely reaching the 20% p-value threshold. Nevertheless, some results are suggestive of fundamental parameters
instability, consistently with the Fed switching to QE policies (de facto liquidity injections).

These early results point towards an unstable behaviour of the Central Bank – if we assume its only behaviour takes the form of a strictly parametrised Taylor rule.

**Diagnostics on structural breaks and Markov Switching** Instead of splitting exogenously the sample according to historical events, in this section we run diagnostics on the full-sample regression to find breaking points. This approach is more data driven, as it makes use of the information contained in the sample to check for breaks and eventually propose the most likely break date(s).

Therefore, we take the models estimated on the full sample and run first a simple CUSUM diagnostic test, then a Chow (1960) test. The latter also retrieves one or more candidate break-points.

The second step is to further unconstrain the data via a Markov Switching estimate. In this last case we adopt Hamilton (1989, 1994) and Murray, Nikolsko-Rzhevskyy, and Papell (2015) approach to our extended sample and only assume it comprises \( k \) discrete states. Then Hamilton (1989) algorithm will provide transition matrices, smoothed probabilities and estimates for each state.

We restrict our analysis to \( k = 2 \), in line with the discussion on the determinacy/indeterminacy regimes at the beginning and at the end of our sample. As aforementioned, pre-Volcker and post-GFC periods yield deeper insight on the functioning of the Federal Reserve monetary policy conduct away from the Great Moderation “steady state”.

CUSUM tests do not report significant fluctuations in the empirical process, meaning that the cumulative sum of the residuals eventually levels off to 0 without significant erratic deviations. Plots of this diagnostic are presented in the Appendix (C.3). The F-Test derived from Chow (1960) points in another direction, though. The output of the test actually reports multiple breaks along the sample, some of which occur with unexpected timing.

Unsurprisingly, when only the most likely date break is requested, four out of six specifications report it around two years into Volcker’s Chairmanship (specifications involving BAA spread start in 1986), much in line with the established consensus. What is more interesting is the picture depicted in the fourth to sixth panels of Fig. (2): these specifications, especially those including the 3-month spread variable, report F-statistics hovering above the threshold for well more than one observation. Investigating more thoroughly this fact, we focus on the number of breaks and their occurrence date, as opposed to the single most likely date as just presented. The results of optimal segmentation of the sample pave the way to the Markov switching estimation below.

**Table 7: Optimal segmentation and break dates**

<table>
<thead>
<tr>
<th>Specification</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. of breaks</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Date 1</td>
<td>1980Q3</td>
<td>1980Q3</td>
<td>1989Q4</td>
<td>1978Q4</td>
<td>1980Q3</td>
<td>1989Q4</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Date 2</td>
<td>1987Q3</td>
<td>1987Q3</td>
<td>2008Q3</td>
<td>1985Q3</td>
<td>1987Q3</td>
<td>2007Q3</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

The first line presents the most likely break admitting only a single one. Third and fourth lines presents break dates when up to 5 breaks are allowed.

Table (7) summarises the analysis on optimal segmentation. It highlights, also, that most of the specifications likely involve more than one single structural break. On top of Volcker’s regime change, one would reasonably expect that the mix of ZLB and unconventional policies would be sufficient to mark an additional break. Surprisingly, it is the case only for Spec. III and VI, whose sample starts only in 1986. A first impression of these results suggests that there might be one or more than one discrete regimes of monetary policy, among which the Federal Reserve switches back and forth. Hence, it is clearly worth pursuing additional insights into these structural breaks with adequate techniques,
Figure 2: F-statistic plots for specifications I to VIII. Solid black line indicates the statistics value, red line marks the significance area at 95%. Time span is rescaled to the interval $[0, 1]$. Individual captions offers most likely date for a singular structural break in the specification.

namely a fully fledged Markov Switching estimation.

Hamilton (1989) provides the algorithm to estimate our specifications with $k$ states, generating also transition matrices and smoothed probabilities to pick the prevailing regime in any date $t$. For every
specification, we allow for the variation of every parameter: in k different states, all parameters are freely estimated, with no constraint posed by other states’ estimates. We make use of the full information set at our disposal, feeding the whole sample to the estimation algorithm.

Table (8) presents estimates for the two state Markov switching model. In contrast to the results of Murray, Nikolsko-Rzhevskyy, and Papell (2015), Specifications I and II – mirroring those of the cited work – find two states complying to the Taylor principle. Once again, Specifications III and IV provide interesting insights.

Specification III yields two states, one strongly complying to the Taylor Principle, the other violating it. In both states, though, liquidity conditions matter for the policy decision but with different magnitudes. Precisely, when the Central Bank fights back inflation (S1), it responds four times more to liquidity conditions, as compared to S2. In the latter, inflation expectations are almost non significant, while the spread on long maturities maps almost entirely into the policy rate. For both states, the Central Bank smooths with the same intensity: this reveals the somewhat minor role of the ZIRP observations. Remarkably, this specification scores the second lowest BIC in the battery of empirical models. As a side note, from the related transition matrix in Table (9) S1 is more persistent than S2.

Specification IV focuses on the shorter term, including the spread on shorter maturities. Results are comparable to those of Spec. III, although magnitudes on liquidity do not vary that much across states. In S1 the Federal Reserve violates the Taylor Principle, ignores the economy slack and puts in place its policy decisions very rapidly, as the smoothing is the lowest across the estimates. On the other hand, in S2 the monetary authority follows more closely a standard, parametrised Taylor Rule, although with a less-than-expected over-reaction to expected inflation. On top of that, S2 exert more attraction than

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5Alternatively, a subset of parameters can be optionally estimated across all regimes, so its estimate is invariant to the prevailing regime.

6Most likely this discrepancy arises from our longer sample and slightly different method employed in the estimation.
$S_1$, as it is possible to devise from Table (9). Interestingly, $S_1$ prevails for short periods of time over the sample (which is longer than that of Spec. III): twice before Volcker, during his Chairmanship, and finally in the early stages of the 2001 and 2008 crises. We might argue, hence, that as economic turmoil sets in the Central Bank turns to damage control and pays more attention to cauterising the financial sector wounds. Once this task is attended to, the Federal Reserve goes back to the regular regime $S_2$.

The panels collected in Fig. (3) depict the prevailing state along the sample for the estimates of Table (8). We also propose the transition matrices for the two estimated states.

Table 9: Transition matrices

<table>
<thead>
<tr>
<th>Spec.I</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>Spec.III</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>Spec.V</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>Spec.VII</th>
<th>$S_1$</th>
<th>$S_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>.9458</td>
<td>.174</td>
<td>$S_1$</td>
<td>.9086</td>
<td>.2399</td>
<td>$S_1$</td>
<td>.9621</td>
<td>.1298</td>
<td>$S_1$</td>
<td>.9455</td>
<td>.0229</td>
</tr>
<tr>
<td>$S_2$</td>
<td>.0542</td>
<td>.826</td>
<td>$S_2$</td>
<td>.0914</td>
<td>.7601</td>
<td>$S_2$</td>
<td>.0379</td>
<td>.8702</td>
<td>$S_2$</td>
<td>.0545</td>
<td>.9771</td>
</tr>
<tr>
<td>Spec.II</td>
<td>$S_1$</td>
<td>$S_2$</td>
<td>Spec.IV</td>
<td>$S_1$</td>
<td>$S_2$</td>
<td>Spec.VI</td>
<td>$S_1$</td>
<td>$S_2$</td>
<td>Spec.VIII</td>
<td>$S_1$</td>
<td>$S_2$</td>
</tr>
<tr>
<td>$S_1$</td>
<td>.9727</td>
<td>.0273</td>
<td>$S_1$</td>
<td>.8423</td>
<td>.0525</td>
<td>$S_1$</td>
<td>.9169</td>
<td>.2021</td>
<td>$S_1$</td>
<td>.8523</td>
<td>.1544</td>
</tr>
<tr>
<td>$S_2$</td>
<td>.0273</td>
<td>.929</td>
<td>$S_2$</td>
<td>.1577</td>
<td>.9475</td>
<td>$S_2$</td>
<td>.0831</td>
<td>.7979</td>
<td>$S_2$</td>
<td>.1477</td>
<td>.8456</td>
</tr>
</tbody>
</table>

Transition probabilities for two states Markov process. Columns are current state, hence conditional on it next state is one of the rows.

This collection of evidences points towards a fundamental instability in parameters of the Taylor rule. Moreover, we retrieved periods with – theoretically – destabilising monetary rules and inflation under control at the same time, once liquidity is included in the decision set of the monetary authority.

Next Section takes stock of these findings and investigates if there is a connection between persistent inflation and monetary rules. For the sake of precision, we restrict our attention to the interplay of monetary policy and realised inflation. To pursue this goal, we present a simple and parsimonious model and carry a horse race with established alternatives.
Figure 3: Markov States: above shaded areas correspond to $S_1$ prevailing over $S_2$, below smoothed probabilities for $S_1$. 
5 Model

This section presents a model to systematize and structure the empirical findings presented in the previous section. This theoretical model builds on Calvo (2016) and relates in spirit with Michaillat and Saez (2015, 2018), B. Diba and Loisel (2017), and M. B. Canzoneri and B. T. Diba (2005) and M. Canzoneri et al. (2008a,b). We augment Calvo (2016) model by fully specifying the supply side of the economy and studying its behaviour in discrete time. Throughout the exposition, our aim is to propose a small deviation from the standard New Keynesian Dynamic Stochastic General Equilibrium (NKDSGE) model that constitutes the core of modern fluctuations theory in macroeconomics (Gali, 2015; Walsh, 2003; Woodford, 2003).

This effort is twofold. First, it allows us to move on known territory, presenting the results in a transparent way and keeping contact with the enormous literature flourished around this class of models. Second, it can be easily implemented in existing theoretical structures with negligible adjustments, making possible a direct test against other models.

5.1 Consumer

Assume an economy with an infinitely-lived representative agent, who works, consumes, and holds money alongside with two types of assets. Hence, total wealth is divided between a liquid bond $B$, cash $M$, and an illiquid bond $X$ whose function serves only intertemporal transmission of consumption. All assets are expire after one period. We assume that the agent is willing to hold $B$ and $M$ because they provide transaction services and therefore utility. In addition, $B$ bonds pay $s$ nominal interest rate, $X$ ones pay nominal interest rate $i$, while money pays no interest and is carried on to next period. Under these assumptions, the utility maximization program of the consumer is structured as follows.

$$\max_{c, m, b, N} E_t \left[ \sum_{s=0}^{\infty} \beta^s \left( u(c_{t+s}) + h(b_{t+s}) + v(m_{t+s}) - g(N_{t+s}) \right) \right]$$

s.t. $c_t + m_t + X_t + B_t = W_t N_t + (1 + s_{t-1}) B_{t-1} + (1 + i_{t-1}) X_{t-1} + M_{t-1}$ (4)

Where we assume additively separable (dis)utilities for consumption, cash, liquid bonds, and hours worked $N$. The inclusion of money and bonds in the utility function is completely equivalent to a cash-in-advance (or rather, liquidity-in-advance) formulation of the model, as in Calvo and Vegh (1996). The intertemporal budget constraint summarises expenditures, allocations and income sources: interests promised to pay in $t-1$, carried on money, and labour income. Moreover, $c$ is the result of aggregating a measure one of differentiated goods via a Dixit-Stiglitz aggregator with elasticity of substitution $\theta$. This also implies that $P$ is the price index of the underlying goods.

Before proceeding to the derivation of the system of first order conditions, it is useful to reformulate the budget constraint to express it in terms of total wealth and real quantities. Therefore, let $D = X + M + B$ be the total wealth held by the consumer. Replacing $X = D - M - B$ in the budget constraint and converting to real terms gives the following budget constraint.

$$c_t + d_t = w_t N_t + \frac{s_{t-1} - i_{t-1}}{1 + \pi_t} B_{t-1} + (1 + r_{t-1}) d_{t-1} - \frac{i_{t-1}}{1 + \pi_t} m_{t-1}$$ (5)

Where $\pi$ is the inflation rate and lower-case indicates real quantities.

With this reformulation, the FOCs system implies the equilibrium conditions in (6).
\[ u'(c_t) = E_t \left[ \beta (1 + r_t) u'(c_{t+1}) \right] \]
\[ \frac{g'(N_t)}{u'(c_t)} = w_t \]
\[ h'(b_t) = E_t \left[ \beta i_{t+1} \frac{i_t - s_t}{1 + r_{t+1}} \right] \]
\[ v'(m_t) = E_t \left[ \beta \lambda_t i_t + \pi_t + 1 \right] \]

(6)

\( \lambda_t \) is the Lagrangian multiplier, the first equation is the usual Euler equation for intertemporal consumption, the second equates marginal cost and benefits of work, and the last two equations govern the allocation decision between liquid bonds \( b \) and real money balances \( m \).

### 5.2 Firms

The production side of this economy is straightforward and assumes a measure one of infinitesimal firms, indexed by \( j \in [0, 1] \). Each firm is embedded with a technology that employs only labour, so that the value added production function is

\[ y_{j_{it}} = A_{t} f \left( N_{j_{it}} \right) = A_{t} N_{j_{it}}^a. \]  

(7)

\( A \) captures the stochastic productivity of the economy and follows a simple AR(1) process, \( a \in [0, 1) \) represents the decreasing returns to scale, and \( N_{j_{it}} \) the individual employment of each firm. As the consumption good results from the CES aggregator, every firm \( j \) faces demand (8), relative to aggregate production, with \( \theta \) being the elasticity of substitution and \( P_{j_{it}} \) the firm’s good price.

\[ y_{j_{it}} = \left( \frac{P_{j_{it}}}{P_t} \right)^{-\theta} y_t. \]

(8)

As in standard NKDSGE models, we assume nominal rigidity à la Calvo (1983):7 every period \((1 - \alpha)\)% of firms are given the chance to update their price, while the remaining share will stick to previous prices. This entices a forward-looking behaviour in firms when they optimise their expected discounted profits by taking into account the duration of their price. Firm \( j \)’s marginal cost is \( MC_{j_{it}} = \frac{W_t}{P_t} A_{t} f' \left( N_{j_{it}} \right) \), whence the expected discounted profits in eq.(9).

\[
\max_{P_{j_{it}}^{*}} \sum_{s=0}^{\infty} \alpha^s Q_{t+s} \left( P_{j_{it}}^{*} y_{j_{it+s}} - MC_{j_{it+s}} y_{j_{it+s}} \right) \\
\text{s.t.} \quad y_{j_{it+s}} = \left( \frac{P_{j_{it+s}}}{P_{t+s}} \right)^{-\theta} y_{t+s}
\]

(9)

Where \( \alpha \) is the Calvo pricing parameter, \( Q_{t+s} \) is the stochastic discount factor between periods \( t \) and \( t + s \) used to weight future profits, and \( P_{j_{it}}^{*} \) is the optimal price chosen by the firm. Factoring in the constraint and solving the program with a symmetry argument gives two results. The first is that firms price with a constant markup over the marginal cost, and second that the optimal price is specified as a function of expected future marginal costs, price index levels and economic activity, as shown in eq.(10). This same equation also presents the inflation dynamic as a AR(1) process.8

7The precise modelling of the nominal rigidity is inconsequential, as the main novelties are in the consumers’ side of the model. Hence, quadratic adjustment costs as in Rotemberg (1982).

8Another interesting derivation of inflation dynamics, close to this one, is presented in Adam and Weber (2018): it includes entry/exit rate and relative experience productivity growth: \( \pi_t = (1 - \delta) \pi_{t-1} + \delta \left( \frac{\bar{y}}{y_t} - 1 \right) + O(2) \).
\[
\frac{P^\ast}{P_t} = \frac{\theta}{\theta - 1} MC_t
\]

\[
\frac{P^\ast}{P_t} = \frac{\theta}{\theta - 1} \sum_{s=0}^{\infty} (\beta\alpha)^s Y_{t+s} \left( \frac{P_{t+s}}{P_t} \right)^{\theta - 1} MC_{t+s}
\]

\[P^1_{t-\theta} = (1 - \alpha) P^\ast_t + \alpha P^1_{t-1}\]

### 5.3 Monetary authority and market clearing

We assume the existence of a monetary authority that operates in two ways in the economy. First, it sets the total amount of liquidity in circulation, namely eq. (11); it must be noticed that the Central Bank does not determine the allocation between cash and liquid bonds, but only the sum of the two, irrespectively of the portfolio composition. For the sake of simplicity, we will also assume that there is a fixed nominal amount of liquidity in the economy, so that

\[Z_t = B_t + M_t\] (11)

\[Z_t = \bar{Z} \quad \forall t\] (12)

Second, it sets the policy interest rate \(s_t\) paid on liquid bonds following some rule: it is useful to think in this context to the link between the Federal fund rate and the interest rate on the shortest-maturity Treasury Bill, as one of the instruments used in the empirical investigation of section 4. In detail, the rule responds simply to inflation expectations:

\[s_t = \gamma E_t \pi_{t+1}\] (13)

This skeletal structure is clearly a simplified Taylor rule, but the values for \(\gamma\) are crucial in showing that an accommodative Central Bank does not necessarily drive the economy down an hyper-inflation spiral, specifically when it takes values smaller than one.

The rationale behind this specification is directly derived by the insights of section 4: keeping under control the liquidity in circulation the Central bank assures that inflation follows a specified path.

Taking eq.(12) and dividing through by the prices level, one can obtain the values for liquidity allocation in real terms, as well as a backward-looking expression for real liquidity depending on current inflation (14).

\[z_t = m_t + b_t \iff z_t = \frac{z_{t-1}}{1 + \pi_t}\] (14)

This last equation, with the market clearing condition \(C_t = Y_t\), closes the model.

### 5.4 System of loglinearised equations and comparison with standard model

In this section we briefly present the system of equations resulting from loglinearising eqs. (6), (10), (13), and (14) around a zero-inflation steady state. To this end, we assume precise functional forms for the utility functions, namely

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\begin{align*}
  u(c) &= \frac{c^{1-\sigma}}{1-\sigma} \\
  h(b) &= \frac{b^\phi}{\phi} \\
  v(m) &= \frac{m^\psi}{\psi} \\
  g(N) &= \chi N^{1+\eta} \frac{1}{1+\eta} \\
  \text{with } 1 &\geq \phi > \psi > 0
\end{align*}

The last assumption implies that the consumer is more sensitive to bonds rather than money, something that is in line with everyday financial decisions. The other functional forms assumed are consistent with more traditional exercises and well settled in the NKDSGE literature.

Once we loglinearise the model we obtain a system of linear equations whose properties can be easily and extensively studied. In the scope of this paper we perform a comparison with the simple 3-equation model presented in Gali (2015), for example.

\begin{align*}
  \hat{y}_t &= \frac{1-\psi}{\sigma} \hat{m}_t + E_t \hat{y}_{t+1} + \frac{1}{\sigma} E_t \pi_{t+1} \\
  \pi_t &= \beta E_t \pi_{t+1} + \kappa (\hat{y}_t - \hat{y}_f) \\
  \left[ \frac{1-\psi}{m^\phi} + m^{\phi-\psi} (1-\psi) \right] \hat{m}_t + \left( \beta\gamma^{-\sigma} m^{1-\psi} \right) s_t - (1 - \phi) \left( 1 + m^{\phi-\psi} \right) \hat{z}_t = 0 \\
  \hat{z}_t &= \hat{z}_{t-1} - \pi_t \\
  s_t &= \gamma E_t \pi_{t+1}
\end{align*}

Briefly describing the system above, eq.(16) is the Euler equation augmented with the real money balances, which affect positively the contemporaneous output gap. Eq.(17) is the Phillips curve of this economy, in line with more classical models, eq.(18) is the money demand function (depending on liquid bonds interest rate \( s \), total real liquidity \( z \)), eq.(19) captures intertemporal changes in total real liquidity, and finally eq.(20) represents the monetary policy rule. We adopt the convention that \( \hat{x} \) is the percentage deviation of \( x \) from its steady state, while all lower-case, unhatted, and time independent variables are steady state values. Moreover, we use \( y^f \) for flexible prices output. To add more details:

\begin{align*}
  \kappa &= \left[ \frac{(1-a) (1-a\beta) a}{a (a + \theta (1-a))} \right] \left[ \frac{1+\eta + a (\sigma - 1)}{a} \right] \\
  \hat{y}_f &= \frac{\eta + 1}{1+\eta + a (\sigma - 1)} \hat{A}_t \\
  y &= \frac{\eta + 1}{1+\eta + a (\sigma - 1)} \hat{A}_t \\
  m &= \left( \frac{y^f}{1-\beta} \right)^{\frac{1}{\gamma}}
\end{align*}

As this approximation of the full model can be fed to software like Dynare, we exploit it to check

---

\footnote{A more detailed walk-through for obtaining this system is provided in the Appendix (D)}
for which calibration sets the models generates a unique and stable equilibrium. In addition, we add two shocks, a real and a monetary one. The former induces a change in the total factor productivity $A$ from its steady-state value, set to 1. The latter is a shock to the monetary rule detailed in eq.(13). These two shocks allow the comparison with the aforementioned standard models, so to perform a horse race and check consistency of our augmented model.

### 5.5 Calibration and IRFs

We calibrate the model setting the values presented in Table (10), taking the most common values used in the literature. It is important to bear in mind that the only restriction involved in the model concerns the exponents of bonds and real balances utility functions. For the “Taylor Principle” of our policy rule, we explore two values: the first is supposed to violate the Blanchard and Kahn (1980) condition, the second is the one estimated in our empirical exercise.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Descr.</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>ret. to scale</td>
<td>.6</td>
</tr>
<tr>
<td>$\beta$</td>
<td>discount rate</td>
<td>.975</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>intertemp. el. of subst.</td>
<td>5</td>
</tr>
<tr>
<td>$\theta$</td>
<td>intratemp. el. of subst.</td>
<td>3.8</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>price duration</td>
<td>.75</td>
</tr>
<tr>
<td>$\psi$</td>
<td>bond el.</td>
<td>.02</td>
</tr>
<tr>
<td>$\phi$</td>
<td>money el.</td>
<td>.65</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Frisch elast.</td>
<td>1</td>
</tr>
<tr>
<td>$\chi$</td>
<td>labour disutility</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Taylor param.</td>
<td>${.5; 1.8}$</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>persistence, TFP shock</td>
<td>.65</td>
</tr>
<tr>
<td>$\rho_{mp}$</td>
<td>persistence, MP shock</td>
<td>.65</td>
</tr>
</tbody>
</table>

Table 10: Calibration for model simulations

First, our calibrated model generates a unique, stable equilibrium for both values of $\gamma$, meaning that sunspot equilibria are ruled out even if the Central Bank reacts passively the inflation expectations of the economy. Second, our model behaves as expected once compared with the 3-equation NKDSGE counterpart (Gali (2015), Chapter III, version with interest rate rule). In fact, under the same calibration the two models show the same behaviour in terms of reactions to shock, as it is possible to see in the next figures. We first compare side-by-side the effects of a technological shock (Fig. (4)), then compare the effects of a monetary policy shock under two regimes for our model (Fig.(5) and Fig.(6)).

As it is possible to see from Fig.(4), a one-standard-deviation, positive shock to total factor productivity produces the same response in our model and in the standard NKDSGE one. This is due to the very same structure of the production side of the two modelled economies. We focus on three common aggregates. Following a TFP shock, in both cases the output gap turns negative, inflation falls, and the reference interest rate falls in response. While this reaction is common across models, magnitudes mark little differences. In particular, while the output gap and inflation falls more in the NKDSGE model, our proposal implies less movement in these aggregates: for inflation and output gap, the response in our model is broadly half on impact. This dampened effect might result from other, structurally different, assumptions made in the consumption and financial blocs of the model, where the TFP shock is dissipated more effectively.

When the models are hit by a monetary policy shock and our model adopts the Taylor Principle,
they generate the IRFs pictured in Fig.(5), again corresponding to common aggregates across the models. The IRFs produce the same profiles in both models, but ours has again a reduced magnitude on impact. For the output gap the effect is roughly the same, for the inflation rate our model responds slightly less than the NKDSGE one.

Looking at the key interest rate it is interesting that our model responds roughly 7 times less than the NKDSGE model to the very same shock, pointing again to a propagation mechanisms that is substantially different and occurs through the financial position of the agent. In addition, our monetary shock is well more persistent in the interest rate, converging back to zero only when inflation levels off, too. Moreover, our proposed policy rule does not factor in the output gap, which drives a feedback loop in the NKDSGE model that is absent in ours. It is worth noticing, though, that a different modelling approach produces the same expected outcomes under the same assumptions regarding the reaction function of the Central Bank.

The two panels included in Fig.(6) show how our model reacts to the previous shocks when the Central Bank does not adhere to the Taylor principle. In this case, we set $\gamma = 0.5$ and expose the behaviour of the endogenous aggregates present in our economy, plus the behaviour of the liquid bond holdings, recovered form the FOC. The first panel, on the left, summarises the consequence of a
monetary policy shock that raises the interest rate $s$ on bonds of 1%.

First off, the output gap moves in the same direction and is affected similarly to the $\gamma > 1$ case. Secondly, inflation responds correctly and after 15 quarters the shock is fully absorbed, without unexpected evolutions of the prices. It is interesting to remark how total real liquidity $z$ and its components, $m$ and $b$, comove in reaction to a monetary policy shock.

Two forces are at work in this case, the inflation effect and the reallocation towards the more remunerative asset. On impact, inflation decreases and then recovers: this path influences real liquidity as future inflation will be higher than today, increasing current liquidity until inflation slightly overshoots the zero-level. This happens in the first five quarters, approximately. When inflation turns positive (extremely close to zero, but still positive) real liquidity peaks and decreases smoothly. In the meantime, the agent adjusts its portfolio of assets profiting of the increased return of the liquid asset. In this sense, the IRF for $m$ mirrors that of $b$, with a reallocation away from money holdings to liquid bonds.

On the quantitative side, it is interesting to remark that under the passive monetary policy regime the economy experiences different magnitudes in the aggregates change. Comparing the behaviour under the two regimes, when the Central Bank is accommodative, output, inflation, and the key interest rate double their impact change, and the latter becomes less persistent.

The left pane summarises the reaction to a TFP shock. As with the aggressive regime, output, inflation and the $s$ interest rate fall. Interestingly, $s$ and $\pi$ slightly overshoot before converging to zero from above from the fifth quarter, roughly. This behaviour is again reflected in the real liquidity and its components: the technological shock doubles the impact on total real liquidity, the interest rate drop triggers a reallocation away from bonds towards cash $m$, until $s$ overshoots and subsequently balances the overall effect on liquidity reshuffling. Appendix (D.3) contains the full IRFs of the model for both cases of $\gamma$ for comparison of the two sets of reactions.

### 5.6 Evaporating liquidity

This model lends itself to an interesting experiment: although nominal liquidity is assumed to be constant at $\bar{Z}$ and real liquidity $z$ moves with the inflation rate, we hit $z$ with a negative shock and study the behaviour of our model, as in eq.(21). Although not orthodox, this is a practical short-cut:
neglecting where that missing liquidity goes physically – and in which proportion money and bonds
are affected – lets us focus on the dynamics of convergence to the steady state. We produce IRFs and
discuss their economic interpretations under the two regimes of monetary policy.

\[ z_t - \epsilon_{z,t} = z_{t-1} - \pi_t, \text{ with } \epsilon_{z,t}/z \approx 10 \]  

(21)

Figure 7: Real liquidity shock: aggressive (dashed) vs accommodative (dotted) policy rules.

Following an abrupt and violent liquidity dry-up the model shows interesting dynamics. The gen-
eral profile/shape is broadly independent of the behaviour of the monetary authority, but substantial
differences in the magnitude arise.

The shock impacts at first money demand \( \hat{m} \), for a given policy rate \( s \), which spikes up. Conversely
the representative agent disinvests in the liquid bond, proportionally more than the missing liquidity.
This results from the preference for money with respect to bonds that assumed in the calibration.

At this stage, the Phillips curve (17) and the IS equation (16) propagate the shock to the rest of the
economy. In particular, the money term in the Euler equation (16) transmits the shock to current output
that spikes as well on impact. Under a passive monetary policy this translates into a limited effect of
the shock on impact, and a degradation in the following quarters. The Phillips curves then squares the
expected inflation with current \( \pi \) and widened output gap. Inflation expectations subsequently drive
the monetary policy decisions which translate into two distinct paths for realised inflation.

The money term in the IS curve is the telling point between the two regimes, together with the path
for liquid bonds, \( b \).

Under both regimes money converges back to the steady state relatively fast, showing that real
liquidity is deeply intertwined with liquid bonds. Most notably, under an accommodative Central Bank real liquidity recovers more rapidly, thanks to a deflation that accelerates the recovery of \( z \) but impedes a quick rebound in output. Interestingly, when the money authority conducts an active policy, the inflation path – contained deflation with slow recovery – turns into persistently low rates, well beyond the case of a passive Central Bank.

The sharp difference in the reaction of the two regimes lies in the severity of the impact and the duration of the recovery. Inflation and output, in particular, show starkly different behaviours: when the Central Bank passively follows inflation expectations a liquidity dry-up triggers a deep recession with a relatively long recovery (more than 30 quarters); the same applies to inflation, too. In a sense, an active monetary stance against a liquidity shock tames the damage and facilitates the recovery, somewhat controlling the effects within the financial sector of the economy.

To sum up, and combining these results with previous information, we suggest that Central Banks complying to the Taylor Principle have a firmer control on contagion when a liquidity crisis hits. According to our stylised model, in fact, active monetary policy helps containing and limiting the damage to the sole financial sector of the economy, with reduced impact and consequence on real activity. The stark difference in the set of IRFs, clearly, lies in the fall of the output gap: it widens under passive monetary policy while it marks a mild recessions under active policy, although duration is similar. As we already remarked, recovery speeds are substantially different.

5.7 Dissecting simulated inflation dynamics

One interesting comparison to carry out involves the inflation dynamics generated by these two models. In particular, keeping in mind what we presented in Section 2, about the different persistence of inflation. It is easy and straightforward to generate abundant time series and hence conduct some econometric exploration. We generate, for each one of the two models 500000 observations, or 125000 years of history: this should assure convergence of the estimators and tight confidence intervals. Of course, the Data Generating Processes of these series is a linear system shocked by AR(1) normal innovations: one should not be surprised that the data generated are also Gaussian, although this goes against the estimated densities of inflation of fig.(20).

On the other hand, it is interesting to check if different models (or regimes) produce different autoregressive properties in inflation under the same calibration and on the basis of the same sequence of shocks. These differences stem from the propagation mechanisms implied by the models, as well as the reactions of the representative agent to such shocks.

For clear reasons, we limit our interest to the global autoregressive properties of the series considering the full sample, which is generated by a single, stable, and well behaved DGP for each case. As we calibrate the persistence of all shocks to \( \rho = .65 \), we expect to find values in this neighbourhood.

We estimate first an \( AR(5) \) for three cases: our model complying to the Taylor Principle, the same violating it, and the standard NKDSGE. Secondly, we set an upper bound on the lags to 120, and pick the optimal lag number minimising the Bayesian Information Criterion.

Table (11) presents the results for an \( AR(5) \). Few things are worth commenting. First off, in all cases the intercept is largely not significant and in a narrow neighbourhood of zero. This is compatible with models that do not include trend inflation or non-zero inflation steady state.

Second, looking at the coefficients on the first lag, we see our expectations confirmed, as all coefficients are tightly close to the calibrated parameter. Interestingly, as we depart from the NKDSGE model to the liquidity model with accommodating Central Bank, the coefficient on the first lag moves away from the calibrated value, downwards. This is interesting because it confirms the consensus that
Table 11: AR(5) estimates on simulated inflation

<table>
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<th>Models</th>
</tr>
</thead>
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<tr>
<td></td>
<td>I</td>
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<tr>
<td>Interc.</td>
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<tr>
<td>(0.00012187)</td>
<td>(0.00022741)</td>
</tr>
<tr>
<td>1st</td>
<td>.63746581***</td>
</tr>
<tr>
<td>(0.00141418)</td>
<td>(0.00141367)</td>
</tr>
<tr>
<td>2nd</td>
<td>−.00347028*</td>
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<tr>
<td>(0.00167708)</td>
<td>(0.00164433)</td>
</tr>
<tr>
<td>3rd</td>
<td>−.00339483*</td>
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<tr>
<td>(0.00167709)</td>
<td>(0.00164435)</td>
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<tr>
<td>4th</td>
<td>−.00471937**</td>
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<tr>
<td>(0.00167709)</td>
<td>(0.00164433)</td>
</tr>
<tr>
<td>5th</td>
<td>−.00795697***</td>
</tr>
<tr>
<td>(0.00141419)</td>
<td>(0.00141367)</td>
</tr>
<tr>
<td>adj. $R^2$</td>
<td>.3992</td>
</tr>
<tr>
<td>BIC</td>
<td>−1032326</td>
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</table>

AR estimates on inflation for liquidity model complying to the Taylor Principle (I), liquidity model violating it (II), and standard NKDSGE (III). We exogenously set the lags to $k = 5$. All models are fed the same sequence of shocks of the same variance, generating 500000 quarterly observations. Significance codes: *p<0.1; **p<0.05; ***p<0.01; SE in parentheses.

A passive monetary authority might let inflation move more rapidly and unconstrained. The flip side of this latter aspect is that, when first lag becomes less relevant, previous ones acquire more weight. Overall, hence, inflation seems to become more persistent when Central Banks do not follow an aggressive Taylor rule.

To substantiate this claim, it is also useful to compare the magnitude of the significant coefficients for model II and models I and III. While for the latter the coefficients quickly approach zero, for model II they become small but remain roughly ten times bigger than those of the other models.

These results point in the direction mentioned in Section 2, where we presented evidence of an increase in inflation persistence.

A more interesting exercise is to compare the optimal lags for an AR($k$) process. This procedure finds that the optimal number of lags for our model with liquidity and Taylor principle (model I) is around 70, for the version without Taylor principle it is around 50, for the standard NKDSGE it is merely 2. Table (12) offers more details on this result.

Table 12: AR(5) estimates on simulated inflation

<table>
<thead>
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<th>Lags</th>
<th>Models</th>
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<tbody>
<tr>
<td></td>
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</tr>
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<td>Optim. lags</td>
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<td>BIC</td>
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Optimal lags for AR process of inflation for liquidity model complying to the Taylor Principle (I), liquidity model violating it (II), and standard NKDSGE (III). Optimal lags are those minimising the BIC. All models are fed the same sequence of shocks of the same variance, generating 500000 quarterly observations.

It must be clear that an optimal lag number does not imply that all regressors are significant. In fact, it only implies that all significant lags are part of the regression, likely a subset of the total lags. With this in mind, it is interesting to appreciate how different the models are in this aspect. Our
model of liquidity with an accommodative monetary authority shows that today’s inflation depends on a long sequence of lags. The number of optimal lags decreases when we let the Central Bank respond aggressively to expected inflation. On the other hand, the standard NKDSGE model produces a process with extremely short memory.

Figure 8: Estimated coefficients on lags, liquidity model with aggressive policy

Figure 9: Estimated coefficients on lags, liquidity model with accommodative policy

Fig.(8) plots the estimates, confidence bands, and p−values\textsuperscript{10} for the 69 optimal lags of model I. Of all lags included, only 21 are strongly significant (±30%); what is interesting is that they are mostly negative in sign.

\textsuperscript{10}For the sake of readability, we do not plot the first lag. All-inclusive plots are in the Appendix (D.4). p−values are rescaled so to fit in the same scale as the coefficients.
Fig. (9) plots the same information for model II. This model features liquidity and an accommodative Central Bank. What is striking is the length of lags deemed relevant and the share of significant ones (±84%), as opposed to model I. As remarked in Table (11), coefficients are greater and all significant at 1% up to the 38th one. These features point toward a high persistence in inflation, something compatible with a Central Bank that, for instance, targets monetary aggregates (Volcker chairmanship) or finds itself short of conventional monetary tools (QE at the ZLB). Nonetheless, this setting does not imply necessarily sunspot equilibria or spiralling aggregates.

More sophisticated versions of this model may be easily developed, as it accommodates additional layers of complexity. For example, one might want to include sticky wages, capital stock, financial blocks in the spirit of the financial accelerator, or even occasionally binding constraints like the Zero Lower Bound. These theoretical devices have been developed as modules for the basic NKDSGE model, with which our model shares many features. The first, natural extension for this model would be the relaxation of the fixed total liquidity in nominal terms, $\bar{Z}$. This would provide the monetary authority an additional tool to carry out its mandate and, most importantly, a framework to study liquidity management.

6 Conclusion
In this paper we first present evidence of the increased inflation persistence over the last years from a purely statistical point of view. This finding holds across a variety of inflation indices and contrast the more volatile behaviour of inflation expectations, as measured by point forecasts and nowcasts.

Opposing the Phillips curve approach, we point toward a policy explanation for this fact. We study the robustness of the standard Taylor rule required in modern New Keynesian Dynamic Stochastic General Equilibrium (NKDSGE) models and cast upon the Central Banks behaviour. We find solid evidence of fundamental parameters instability across a wide set of econometric techniques and methods, as well as the presence of allegedly multiple monetary policy regimes over the Federal Reserve policy history.

On top of this parameter instability, we also find that the Taylor Principle, warranting uniqueness and stability in NKDSGE models, is violated in some specifications of the monetary authority behaviour. Nevertheless, this violation does not bring about degenerate occurrences of sunspot equilibria, with hyper-inflation or deep disinflation episodes.

To connect the inflation increased persistence, we tweaked a parsimonious and well-known DSGE model adding a more refined financial sector. We included liquid and illiquid assets, adapting the monetary response to liquidity conditions in the economy. This simple model proves stable and engenders a unique equilibrium independently of the Taylor principle. Hence, our model encompasses regimes of accommodative and aggressive monetary policy, at the cost of a little modification of the workhorse model.

We also compare our model (in its two regimes) with the workhorse of the New Keynesian literature. Under the same calibration, our model closely maps the NKDSGE according to the impulse response functions metrics, as reasonable disparities arise only in the magnitude dimension. Moreover, we reproduce the methodology we applied to actual inflation series. We find that our model, calibrated as to violate the Taylor Principle, generates a sluggish inflation, way more persistent as its current value depends on a greater number of lags. Hence, it seems to confirm (or provide theoretical ground) our intuition concerning the connection between monetary policy rules and inflation dynamics.

This paper contributes to the discussion on the interplay between monetary policy rules and inflation...
tion dynamics. This connection, although suggestive, leaves ample room for additional investigation. In particular, we restricted our focus solely on the interaction of the Central Bank and actual inflation. The previous findings, thus, beg for further inquiry.
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### A Data sources and transformations

Table 13: Data details for the US

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Footnoted series start in 2007Q1.
B  Inflation dynamics

This section presents Figs. (10) and (11), which are plots like (1) but estimated with shorter windows width, ten and five years respectively.

Figure 10: AR\((k^*)\) estimates of \(\rho_1\) – 10 years window. Solid black line is the point estimate on \(\rho_1\), red bands mark 2SE area. In blue, polynomial LOESS fit conveys the overall trend from local observations.
Figure 11: AR \((k^*)\) estimates of \(\rho_1\) – 5 years window. Solid black line is the point estimate on \(\rho_1\), red bands mark 2SE area. In blue, polynomial LOESS fit conveys the overall trend from local observations.
C Empirical appendix

C.1 Plotting collected data

Figure 12: Standard Taylor Rule variables

Figure 13: Measures of output slack
One quarter ahead inflation forecasts

Current period inflation forecasts

Figure 14: Inflation forecasts

Figure 15: Inflation nowcasts
Figure 16: Spreads on short and long maturity assets: S&P500 vs 10y BAA corporate bond returns index

Figure 17: Measures of revised inflation, with and without food and energy items. Historical values after latest revision available.
Figure 18: Average point forecasts for several inflation indexes from the Survey of Professional Forecasters - Philadelphia Federal Reserve Bank.

Figure 19: Interquartile range in cross section point forecasts for several inflation indexes from the Survey of Professional Forecasters - Philadelphia Federal Reserve Bank.
Figure 20: Estimated kernel densities for three inflation measures.
C.2 Full sample regression: residuals

Fig.(21) plots the residuals generated from the regression on the full sample. As OLS sort of averages over the full sample, sudden and ample fluctuations in the residuals point to observations where the model underperforms. This occurs typically in the late '70s, late '80, and around the GFC period.

Figure 21: Residuals plot for the eight specifications. Black solid line depicts single residuals as time series, red bands contour the 2-SDs area around zero, the expected – and empirical – residuals mean.
C.3 CUSUM tests plots

This section presents the CUSUM plots for the eight specifications of the Taylor rule studied in Section (4). The solid black line marks the cumulative sum of residuals, whilst the red lines define the significance areas, in which the residuals sum signals a change in the underlying data generating process.
D Model appendix

D.1 Obtaining the system of equations

We combine the equations presented in the body of the paper so to obtain the system of equations that will be later loglinearised and fed to Dynare for simulations.

The first target is the augmented Euler equation. The version proposed in the paper includes real money balances on top of the usual terms. Take the first intertemporal FOC from the consumer utility maximisation program as the starting point:

\[ u'(c_t) = E_t \left[ \beta \left(1 + r_t \right) u'(c_{t+1}) \right] \]

recall the Fisher equation \((1 + r_t) (1 + \pi_{t+1}) = (1 + i_t)\) and replace \(r_t\)

\[ u'(c_t) = E_t \left[ \frac{\beta u'(c_{t+1}) (1 + i_t)}{1 + \pi_{t+1}} \right] \]

\[ = E_t \left[ \frac{\beta u'(c_{t+1})}{1 + \pi_{t+1}} + \frac{\beta u'(c_{t+1})}{1 + \pi_{t+1}} i_t \right] \]

Now recall the condition on marginal utility of real money balances, \(v'(m_t)\), and employ it to define the nominal interest rate, \(i_t\):

\[ v'(m_t) = E_t \left[ \beta \lambda_{t+1} i_t \right] \]

\[ i_t = v'(m_t) E_t \left[ \frac{1 + \pi_{t+1}}{\beta \lambda_{t+1}} \right] \]

moreover \(\lambda_{t+1} = \frac{u'(c_{t+1})}{E_t}\)

Turning back to the Euler equation, plug in the nominal interest rate relation just recovered:

\[ u'(c_t) = E_t \left[ \frac{\beta u'(c_{t+1})}{1 + \pi_{t+1}} + \frac{\beta u'(c_{t+1})}{1 + \pi_{t+1}} v'(m_t) \frac{1 + \pi_{t+1}}{\beta u'(c_{t+1})} \right] \]

which rearranges in

\[ = E_t \left[ \frac{\beta u'(c_{t+1}) (1 + \pi_{t+1})}{1 + \pi_{t+1} + v'(m_t) \frac{1 + \pi_{t+1}}{\beta u'(c_{t+1})}} \right] . \]

This last expression is then loglinearised to obtain equation (16).

To condense the money equation start with the last two relations in (6):
\[ v'(m_t) = E_t \left[ \beta \lambda_{t+1} \frac{i_t}{1 + \pi_{t+1}} \right] \]

\[ i_t = v'(m_t) E_t \left[ \frac{1 + \pi_{t+1}}{\beta \lambda_{t+1}} \right] \]

plug this result into \( h'(b_t) \)

\[ h'(b_t) = E_t \left[ \beta \lambda_{t+1} \frac{i_t - s_t}{1 + \pi_{t+1}} \right] \]

\[ \Rightarrow h'(b_t) = E_t \left[ \beta \lambda_{t+1} \frac{\beta \lambda_{t+1} s_t}{1 + \pi_{t+1}} - \frac{\beta \lambda_{t+1} s_t}{1 + \pi_{t+1}} \right] \]

exploit the fact that \( \lambda_{t+1} = u'(c_{t+1}) \) and \( b_t = z_t - m_t \) to obtain

\[ h'(z_t - m_t) - v'(m_t) = E_t \left[ -\frac{\beta s_t u'(c_{t+1})}{1 + \pi_{t+1}} \right] \]

Setting this equation to its steady state and using a first Taylor approximation generates equation (18) in the text.

The backward dependence of real liquidity (14) results as follows:

\[ Z_t = Z = M_t + B_t \]
\[ M_t = P_t m_t \quad B_t = P_t b_t \]
\[ \ddot{Z} = P_t m_t + P_t b_t \]
\[ \ddot{Z}_{t-1} = (m_t + b_t) \frac{P_t}{P_{t-1}} \]
\[ z_{t-1} = z_t (1 + \pi_t) \]
\[ \Rightarrow z_t = \frac{z_{t-1}}{1 + \pi_t} \]

Concerning the Phillips curve, it remains unchanged from traditional New Keynesians models and derives from the use of equations (10). The output gap it includes results from the comparison to the flexible prices version of the model. Other relations do not need further manipulation.

### D.2 Loglinearisation

The model is loglinearised around a zero inflation steady state as in the early New Keynesian models. This assumption yields the remaining variable values in the long run and without shocks. We employ directly the functional forms from (15).

**Steady state values**

Euler equation / IS curve at the steady state:

\[ u'(c_t) = E_t \left[ \beta (1 + \pi_t) u'(c_{t+1}) \right] \]

\[ e^{-\sigma} = m^{\Psi-1} + \beta \frac{e^{-\sigma}}{1 + \pi} \]

\[ m^{1-\Psi} = \frac{e^\sigma}{1 - \beta} \]

Phillips curve: \( y = y' \).
Money demand:

\[ h' (z_t - m_t) - v' (m_t) = E_t \left[ -\frac{\beta s_t u' (c_{t+1})}{1 + \pi_{t+1}} \right] \]

\[ s_t = \gamma \pi_{t+1} \]

\[ (z - m)^{\phi-1} - m^{\psi-1} = -\frac{\beta}{s} c^{-\sigma} \]

\[ = 0 \]

\[ (z - m)^{\phi-1} = m^{\psi-1} \]

\[ z - m = m^{\pi/z} \]

\[ z = m + m^{\pi/z} = m \left( 1 + m^{\pi/z} \right) \]

Remarkably, if \( \psi = \phi \), so that the agent is indifferent between money and liquid bonds, \( z = 2m \) as in the log preferences case.

**Loglin**

Loglin for liquidity law of motion:

\[ z_t = \frac{z_{t-1}}{1 + \pi_t} \]

\[ \Rightarrow \hat{z}_t = z_{t-1} - \pi_t \]

This equation does not pin down the steady state value for \( z \), since it results from the sum of real money balances, \( m \), and liquid bonds, \( b \).

\[ \hat{b}_t = \frac{z}{z - m} \hat{z}_t - \frac{m}{z - m} \hat{m}_t \]

Loglinearised Euler equation:

\[ c_t^{-\sigma} = E_t \left[ \beta c_{t+1}^{-\sigma} \left( 1 + \frac{m_{t+1}^{\phi-1}}{\hat{c}_{t+1}} \right) \right] \]

Taking logs and first derivatives - drop \( E_t \) for convenience - and rearrange:

\[ -\sigma \epsilon_t = (\psi - 1) \hat{m}_t + (-\sigma \epsilon_{t+1} - \pi_{t+1}) \]

\[ \hat{c}_t = \frac{1 - \psi}{\sigma} \hat{m}_t + \hat{c}_{t+1} + \frac{1}{\sigma} \pi_{t+1} \]

Loglinearised money demand:

\[ h' (z_t - m_t) - v' (m_t) = E_t \left[ -\frac{\beta s_t u' (c_{t+1})}{1 + \pi_{t+1}} \right] \]

\[ (z_t - m_t)^{\phi-1} - m_t^{\psi-1} = E_t \left[ -\frac{\beta u_t c_{t+1}^{-\sigma}}{1 + \pi_{t+1}} \right] \]

\[ (1 + \pi_{t+1}) (z_t - m_t)^{\phi-1} = (1 + \pi_{t+1}) m_t^{\psi-1} - \beta c_t^{-\sigma} \]

Focus first on the left-hand side of the above equation and drop the steady state terms – which will cancel out eventually:

\[ \ln (1 + \pi_{t+1}) + (\psi - 1) \ln (z_t - m_t) \Rightarrow \pi_{t+1} + (\psi - 1) \left( \frac{z_t - m_t}{z - m} \right) - \frac{m_t}{z - m} \hat{m}_t \]

Now replace the steady state value for \( z \) found in the money demand above and plug in the previ-
ous equation, factoring out the common terms, we obtain

\[
\pi_{t+1} + (\psi - 1) \left[ m^{1-\psi} (z_t - m \hat{m}_t) \right]
\]

\[
\pi_{t+1} + (\psi - 1) \left[ \hat{z}_t + (\hat{z}_t - \hat{m}_t) m^{\frac{\psi}{1-\psi}} \right]
\]

Turning to the right-hand side of the previous equation, we know that at the steady state and in logs it equals \( \ln (m^{\psi-1}) \), as the term involving \( s \) collapses to 0. Furthermore, we can break down the loglinearisation in two chunks, the first yielding simply

\[
\pi_{t+1} + \frac{\psi - 1}{m^2} \hat{m}_t
\]

and the second one not more difficult. Namely, loglinearising with respect to \( c \) yields 0, as it contains \( s = r \pi = 0 \) at the steady state; approximation for \( s \) gives

\[
-\beta \frac{c - \psi}{m^{\psi-1}} \hat{s}_t
\]

Gathering all pieces together gives, after slight rearrangements:

\[
\left( \frac{1 - \psi}{m^2} \right) \hat{m}_t + \beta e^{-\sigma} m^{1-\psi} \hat{s}_t - (1 - \psi) \left[ \hat{z}_t + (\hat{z}_t - \hat{m}_t) m^{\frac{\psi}{1-\psi}} \right] = 0
\]

From which we can recover eq.(18) in the text.
D.3 Full IRFs

In this section we plot the full set of impulse response function produced by our model with liquidity. We differentiate the two monetary policy regimes as active $\gamma = 1.8$ or passive/accommodative $\gamma = .5$. $y_{-gap}$ is the output gap, $pi$ the quarterly inflation rate, $s$ the policy interest rate on liquid bonds, $m, z,$ and $b$ are real money holdings, total real liquidity, and liquid bond holdings, respectively.

**Figure 22:** Full IRFs in our model of liquidity: TFP (left) and MP (right) shocks, active monetary policy regime.

**Figure 23:** Full IRFs in our model of liquidity: TFP (left) and MP (right) shocks, accommodative monetary policy regime.
D.4 Simulated inflation: robustness, comparison with Smets and Wouters (2007)

Table 14: AR(5) estimates on simulated inflation

<table>
<thead>
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<th>Lags</th>
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<tr>
<td>Optim. lags</td>
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<td>II</td>
<td>III</td>
<td>IV</td>
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<td>−597733.3</td>
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</tbody>
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Optimal lags for AR process of inflation for liquidity model complying to the Taylor Principle (I), liquidity model violating it (II), standard NKDSGE (III), Smets and Wouters (2007) (IV). Optimal lags are those minimising the BIC. All models are fed the same sequence of shocks of the same variance, generating 500000 quarterly observations.
Figure 24: Estimated coefficients on lags, Smets and Wouters (2007) – 500k observations. Above panel: excluding first lag; below panel: all lags.
Figure 25: Estimated coefficients on lags, our model with liquidity. Above panel: aggressive monetary policy $\gamma > 1$; below panel: passive monetary policy $\gamma < 1$. 
E NKDSGE to its extrema

This section explores two extreme calibrations for the standard workhorse NKDSGE. We test two limiting cases, namely a radically aggressive Central Bank ($\gamma = 180$) and an almost-accommodative one ($\gamma \to 1^+$. The two cases produce interesting behaviours, especially for the aggressive Central Bank: this latter puts in place a monetary conduct that chokes any reaction in the other aggregates. While the left panels of figs. (26) and (27) picture a scenario close to the usual one, the right panels depict an economy with no fluctuations. The TFP shock in particular, triggers only a movement in the policy rate that absorbs and prevents the transmission of the shock to other variables.

Figure 26: Passive (left) and extremely aggressive (right) Central Bank policy: 1% rate hike

Figure 27: Passive (left) and extremely aggressive (right) Central Bank policy: TFP shock