Systemic Risk from Interbank Credit Markets?

A Contribution to a Resilient Financial System

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Abstract

This theoretical model analyzes the impact of interbank credit market dynamics on systemic risk in the economy. Starting with a single bank's balance sheet, which includes interbank activities, we derive general portfolio equilibria in financial markets. Based on these static equilibria, a stochastic model of interbank market dynamics is introduced, which can identify a dynamic instability. We define financial market resilience as the probability of a stable adjustment process to portfolio equilibrium. A change in the volatility of reserve flows, which we often observe when central banks tighten monetary policy, may threaten the resilience of interbank markets and increase the probability of the market to fall into a regime of unstable dynamics. Thus, we stress that monetary policy could incidentally reduce financial stability, especially under a monetary policy regime-switch from an expansionary to a contractionary policy. When switching the regime, policymakers should be aware of a potential reduction in interbank credit market resilience and the consequences for financial stability.

JEL classification code: E44, E52, G11, G21
JEL keywords: Financial Markets, Interbank Lending, Systemic Risk

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1 Introduction

In September 2008, the U.S. investment bank Lehman Brothers had to apply for insolvency due to write-offs in the course of the financial crisis. This default of one single bank first caused write-offs in interconnected banks' balance sheets before spreading not only within the financial system of the U.S. but also around the globe. Since then, there has been increasing interest in researching the role of interbank markets and the consequences of interbank activities. The observed enormous domino effects of bank defaults have generated growing interest in financial interconnectedness in interbank networks and contagion channels (see, amongst others, Heider et al., 2015; Mistrulli, 2011; Vollmer and Wiese, 2014; Rünstler, 2016), which can ultimately result in a systemic threat (cf. Acemoglu et al., 2015). 2018 marked the tenth anniversary of the Lehman Brothers insolvency. Although the consequences of the crises have led to a vast literature on how to identify, measure, and alleviate systemic risk as well as first real world applications and implementations in the context of macroprudential policies, the banking system is still not resilient.

While the definition of systemic risk is not clear-cut, multifarious, difficult to capture (Hansen, 2014), and has changed over time, the importance of countermeasures is a given. The literature mainly refers to the definition of the ECB (2009), which refers to systemic risk as the "*risk of threats to financial stability that impair the functioning of a large part of the financial system with significant adverse effects on the broader economy.*" (see also Di Cesare and Rogantini Picco, 2018). Early surveys by De Bandt and Hartmann (2000) up to a recent contribution of Di Cesare and Rogantini Picco (2018) cover a vast range of, e.g., systemic risk indicators, empirical studies and macroprudential policies but leave a blank space where the impact of interbank credit markets on systemic risk should be covered theoretically. While research on systemic risk is clearly dominated by empirical studies (Silva et al., 2017), we contribute to the theoretical background analysis of systemic risk factors in the financial system using a portfolio management approach with a focus on interbank credit markets.

What can trigger financial crises and how does systemic risk evolve? We refer to the "capital view" (Freixas et al., 2015), which states that the build up of systemic risk is endogenous to the financial system. Systemic risk builds up over time, stemming, e.g., from asset price bubbles, excessive risk-taking by financial institutions, balance sheet channels, or market and funding illiquidity. Microprudential policy is unable to identify this endogenous risk but without appropriate policy responses, systemic risk builds up and financial crises are realized.

We shed light on the role of interbank credit markets with respect to systemic risk. According to Paltalidis et al. (2015) the interbank loan market is one source of systemic risk, while Iori et al. (2006: 526) name interbank lending as "*one form of safety net for individual banks*". This trade-off between mutual insurance and systemic risk is also the focus of this work. Systemic risk can arise when information or news shocks reduce the interbank credit supply, which can

ultimately lead to a complete dry-out of the interbank credit market. During the crisis, this interbank illiquidity put a stranglehold on the private sector and led the ECB to assume the role of lender of last resort in the post-crisis years. Furthermore, the ECB adopted unconventional monetary policies to avoid the transmission of its policies to be stuck in the interbank markets, and acted as interbank intermediary between surplus and deficit banks (cf. Giannone et al., 2012).

Having decreased the policy rates to the zero-lower bound in the aftermath of the crises, central bank's policymakers are recently considering raising policy rates again and launching a contractionary monetary policy regime. Can this regime-switch be regarded as a smooth return to "normal" or is it a risk-bearing operation? And how can this switch trigger financial market instability?

For instance, the ECB has decided to end net asset purchases in December 2018 and considers to tighten monetary policy, although uncertainties relating to financial market volatility have gained in importance (ECB, 2018). Figure 1 shows the monthly relationship between the monetary policy rate and a volatility index for the EuroStoxx (VSTOXX) over the period 2000 - 2018. Overall, a positive correlation between the monetary policy rate and asset price volatility can be assumed to hold. If this is true, a contractionary monetary policy would go hand in hand with higher asset price volatility and hence a higher volatility in reserve flows in banks' balance sheet management. This would increase the probability of an unstable adjustment process in the interbank market. Consequently, we show that contractionary monetary policy could decrease interbank market resilience. This asymmetric reaction scheme is described in more detail in section 4. When switching the regime, policymakers should be aware of a potential reduction in interbank credit market resilience and the consequences for financial stability.

So far we have been mainly able to identify two strands of related theoretical models on banks' liquidity management, the role of interbank lending, and systemic risk.

A first strand deals with static, partial equilibrium microeconomic models of financial frictions in the interbank market. Freixas and Jorge (2008) consider a real shock, which induces a need for liquidity by individual banks. The liquidity shortage can be settled either by the central bank or by interbank market borrowing, which is characterized by asymmetric information. Freixas and Jorge (2008) show that these frictions in the interbank market cause an equilibrium with rationing in the credit market. Heider and Hoerova (2009) examine the functioning of unsecured and secured interbank markets in the presence of credit risk in a portfolio choice model. Hauck and Neyer (2014) analyze the impact of frictions in the European interbank market on bank loan supply and suggest to reverse the intermediary function assumed by the ECB by reduced interbank market participation costs or reduced liquidity costs, which arise from transactions in the interbank market. Bucher et al. (2017) model a bank run and the respective liquidity management of a single bank, which maximizes profits and minimizes liquidity costs under financial imperfections in the interbank market. Biondi and Zhou (2017) develop an agent-based model of the interbank market

Monetary Policy Stance (MRO) (X) and VSTOXX (Y)
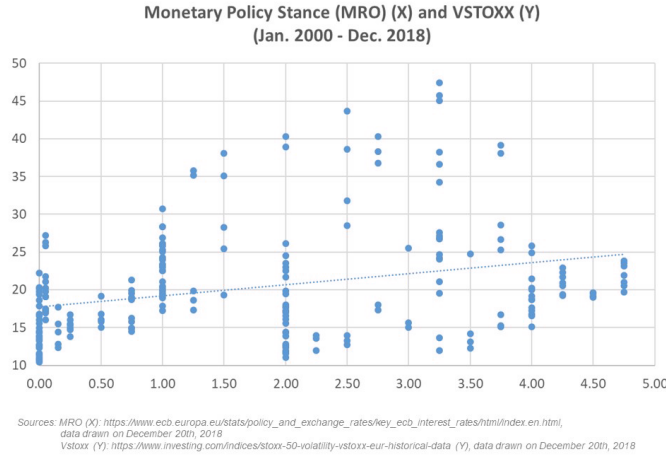**(Jan. 2000 - Dec. 2018)**

Figure 1: Relation of central bank policy rate and volatility

in a bilateral transaction analysis.

The second strand relates to DSGE modelling on the macro-level of systemic risk and deals with the intertemporal effects of an exogenous shock.

However, the two strands are rarely combined. For instance, Allen and Gale (2004) develop a general model of financial markets and financial intermediaries with complete and incomplete contracts as well as markets that are subject to an aggregate shock. Diamond and Rajan (2006) suppose that the use of real demand deposits (e.g., foreign exchange denominated) to refinance banks can lead to illiquidity or even insolvency, while the negative effect can be mitigiated if banks are refinanced with nominal deposits. Diamond and Rajan (2012) show that deposits restrict banks' investments in illiquid assets, and deal with bank runs as shocks. However, "*a truly synthesized approach is thus far still missing*" (Freixas et al., 2015); also to our knowledge none of those hybrid models takes interbank market dynamics into consideration.

Therefore, we examine the dynamic effects of an individual bank's interbank lending activities within its portfolio management, which leads the interbank market into a stable or unstable equilibrium. In our theoretical analysis, the interbank market is a well-functioning element in the resilient financial system, provided everything goes as expected, but an exogenous shock can lead in an extreme case to a complete shut down of the interbank credit market. We present a theoretical framework, which identifies the importance of interbank lending dynamics for the resilience of the financial system. Our results suggest the importance of safeguarding the smooth provision of interbank credits over time to ensure a functioning financial system.

A wide range of macroprudential policies and supervision was devised to

avoid a repetition of disastrous events. Amongst other things, they were designed to safeguard the efficient liquidity transfer in the interbank market (see, e.g., Furfine, 2001, Acharya and Yorulmazer, 2008). While interbank markets in general allow for risk-sharing across banks (Bhattacharya and Gale, 1987), interbank exposures can also result in systemic risk. This was also observed during the financial crisis, when interbank market rates increased significantly and transaction volumes tended towards zero. Macroprudential indicators of systemic risk distinguish between the cross-sectional dimension (contribution of individual banks) and the time dimension (procyclicality of systemic risk) (Borio, 2003). Can macroprudential policies avoid systemic risk and improve the financial system's resilience? To answer that question, we acknowledge the importance of the interbank credit market and add to the discussion on the efficacy of macroprudential policies by combining the cross-sectional with the time dimension in an illustration of the dynamics in the interbank credit market as a potential source of systemic risk.

2 Private banks

In this model, the (domestic) banking sector consists of many competing banks. Banks operate in competitive markets for loans, deposits, and reserves (cf. Bianchi and Bigio, 2017). The banking sector can be divided into two groups. Each group is represented by one bank, i for one group or j for the other group. We condense our descriptions to the activities of the representative bank i of a group with a liquidity surplus and start with the portfolio management of bank i.

2.1 Bank i's assets

For each bank i, which holds a liquidity surplus, we want to model the optimal portfolio strategy. Therefore, we look at the asset side of the balance sheet, which records the use of funds. In the balance sheet of bank i (BS_i) we distinguish between three groups of assets: central bank reserves, interbank credits, and bonds. First, we introduce these assets and their respective properties. Each asset has a distinct risk-return combination, where risk refers to the effects on portfolio risk. For instance, central bank reserves in general reduce portfolio risk but generate costs; interbank credits and securities in general increase portfolio risks but also generate returns. The following sub-sections describe the function of each asset in the portfolio of the representative bank i.

Liquid asset of Central Bank reserves First, the individual bank i decides on the amount of central bank (CB) reserves it wishes to hold (M_{CBi}) to cover its differentiated need for riskless liquidity. The CB provides credits via refinancing operations (simplified to "reserves" or CB money M_{CB}), which reduce the portfolio risk of bank i's balance sheet ($\sigma_{M_{CB}}$). Bank i's need for CB reserves consists fulfilling its minimum reserve requirement, imposed by the CB,

and so-called "autonomous factors", such as cash demanded by its customers. Depending on its current stock of CB reserves, bank i can demand CB reserves, which are provided at the official interest rate i_R. Bank i can also deposit excess liquidity overnight at the CB. The market for CB reserves can be regarded as the primary money market for the representative bank (e.g., Affinito, 2013). We assume that CB reserves (M_{CB}) are always available and thus, portfolio risk decreases with increasing CB reserves in the bank's balance sheet. In this model, we focus on the role of interbank market activities with respect to financial stability. We hence abstract from the bank's lending to non-banks and the respective liabilities to non-banks, which implies an abstraction from minimum reserve requirements as well as "autonomous factors". Consequently, bank i holds highly liquid CB reserves for its individual portfolio management purposes only.

If we look at the ECB, several instruments are used to manage the Euro area banking sector's liquidity (see ECB, 2011 for a detailed description of its instruments), which can be reduced to forward guidance as well as three key interest rates. First, the main refinancing operation (MRO), which traditionally satisfies approximately 74% of the liquidity needs of the Euro area's banking sector (ECB, 2002), and takes place weekly. Second, marginal lending facilities provide overnight liquidity from the CB at higher costs than those of the MRO, and finally, the deposit facility represents an overnight deposit of our representative bank i, held at the CB. As financing rate, we refer to the MRO due to its main application. Moreover, we abstract from the rigidities of the MRO to keep the analysis traceable and refer to the bank's deposits at the CB as a CB credit repayment.

Investment opportunity 1: Interbank Credits

In case of a liquidity surplus, the bank can use its funds for different purposes. Each bank takes an investment decision on credit provision in the interbank market. However, they have different characteristics with respect to their return, i.e., interest rate and risk. Generally, both investments earn a positive return and bear some risk.

First, bank i can provide its excess liquidity to other banks via an **interbank credit**. The amount of interbank credits given from bank i to bank j is denoted by CR_{IB_i}, where the first subindex refers to the credit providing bank (credit from bank i), and the second to the loan receiving bank (credit to bank j). As all j banks are identical all credits given by bank i to the j banks are identical such that we can simplify the notation to $CR_{IB_{ij}} = CR_{IB_i}$. Bank j's demand of an interbank credit, i.e. borrowing in the interbank market is denoted as CR_{IB}^D. Real interbank markets are characterized by over-the-counter trade, in which contracts and terms of conditions are negotiated individually by the lending and borrowing banks (e.g. Vollmer and Wiese, 2014; Bianchi and Bigio, 2017). The terms of loan provision foresee an individual interest rate, which in this model is denoted by i_{IB}. Interbank loans are not insured and often uncollateralized (Furfine, 2001), which increases the lender's risk exposure. Furthermore, the

lending bank does not have to hold minimum reserves on the interbank liquidity provided, which increases the associated risk. We examine the flow mechanism of interbank credits in detail in the following section *3.2.*

We focus on financial stability and the correponding role of interbank market activities. Therefore, we abstract from bank's lending to non-banks and the resulting liabilities to non-banks. Consequently, bank i 's reserve holdings no longer constitute official minimum reserves on the liabilities to non-banks but are a chosen liquidity back-up for portfolio management purposes.

Investment opportunity 2: Bonds In addition to providing interbank credits, bank i can invest in a number of different assets, which are represented by domestic government bonds (B_i). Bank i decides to hold a number of government bonds B_i. Bonds are defined in the standard way. They have an infinite life time, provide a given fixed interest payment of one unit, and can be traded at current market price P_B. Thus, we obtain the standard relation between bond prices and returns i_B, $P_B = \frac{1}{i_B}$. Consequently, their value in bank i's balance sheet equals $P_B B_i$. Government bonds are regarded as a low-risk investment (σ_B) with a presumed relatively low return. We abstract from further assets, such as foreign bonds, stocks, fiduciary assets, or tangible assets, to maintain our focus on interbank market activities and not dilute the analysis with other assets of minor importance.

2.2   Bank i's balance sheet

Having considered single entities of the asset side of the balance sheet of an individual bank i, the complete balance sheet is structured as follows.

<div align="center">

Bank-i

assets	liabilities
(i) balance with central bank M_{CB_i}	
(ii) (interbank) lending to banks CR_{IB_i}	(iv) (interbank) liabilities to other banks L_{IB_i}
(iii) bonds $P_B B_i^D$	(v) capital EK
balance sheet total BS_i	balance sheet total BS_i

</div>

The simplified asset side shows the use of funds and contains the following single entities (i) - (iii): Balance with the CB, lending of bank i in the interbank market, as well as the value of bank i's investment in a representative asset of bonds. The simplified liability side depicts the sources of funds, which are reduced to liabilities to other banks (iv), such as interbank borrowing.

It is important to note that interbank credit activities are usually aggregated to either a net asset or a liability position. However, we account for both positions in the balance sheet to keep the analysis traceable also for borrowing banks' j. Principally, bank i's balance sheet records the loans given, which means bank i's lending in the interbank market (CR_{IB_i}) (ii), while interbank credits taken are recorded in position (iv). Therefore, in this model, bank i's liquid assets stem either from the CB or from the interbank market, where L_{IBi}

is bank i's borrowing in the interbank market. Furthermore, the balance with the CB (i) constitute only a voluntary reserve, which is held to safeguard bank i's solvency.

In our portfolio choice model we need to analyze the asset side of bank i's balance sheet. Hence, equation 1 describes the sum of the simplified asset side of bank i' s balance sheet, which equals the balance sheet total.

$$M_{CB_i} + CR_{IB_i} + P_B B_i^D = BS_i \qquad (1)$$

2.3 Bank i's optimal portfolio choice

The commercial bank tries to choose an optimal portfolio structure that maximizes expected utility of profits earned from this portfolio.

Bank i's portfolio profits and risks
The bank's portfolio profits π_i are simply the sum of elements that earn minus the cost-elements. In particular, a bank i earns from lending to other banks $(i_{IB}CR_{IB_i})$ and holding government bonds $(P_B B_i^D)$, while the portfolio generates costs for holding CB reserves $(-i_R M_{CB_i}^D)$. We do not assume any costs of portfolio management or bank production of services.

$$\pi_i = -i_R M_{CB_i} + i_{IB}CR_{IB_i} + i_B B_i^D \qquad (2)$$

However, banks not only earn profits with their portfolio, they also take risks. Each asset stands for a different risk-return characteristic. However, in principal, all earning assets (CR_{IB_i}, B_i^D) are defined by positive contributions to profits, but also to the portfolio risk σ_i; each in its own way.

$$\sigma_i \left(M_{CB_i}, CR_{IB_i}, B_i^D \right), \qquad (3)$$

$$\frac{d\sigma_i}{dM_{CB_i}} < 0, \quad \frac{d\sigma_i}{dCR_{IB_i}} > 0, \quad \frac{d\sigma_i}{dB_i^D} > 0$$

Bank i's expected utility of asset portfolio
Banks are risk averse and maximize the expected utility of the profits of their portfolio.

$$V_i = V_i(\pi_i, \sigma_i(M_{CB_i}, CR_{IB_i}, B_i^D)) \qquad (4)$$

Bank i's optimization problem
The portfolio optimization problem can now be described as

$$\begin{aligned} \max \quad &: \quad V_i = V_i \left(\pi_i, \sigma_i(M_{CB_i}, CR_{IB_i}, B_i^D) \right) \qquad (5) \\ s.t. \quad &: \quad M_{CB_i} + CR_{IB_i} + P_B B_i^D = BS_i \end{aligned}$$

Bank i's asset demand functions:

From the bank's optimization problem we can derive the bank's asset demand function for portfolio optimization with a demand for CB reserves, the demand for the asset of an interbank credit supply as well as the bank's demand for bonds as described in the following proposition.

Proposition 1 *Bank i's optimal asset demand functions: Problem (5) and the respective FOC implicitly define bank i's asset demand functions for the three groups of assets[1]*

$$
\begin{array}{llll}
\text{reserves (i)} & \text{CB money} & M_{CB_i} = m_{CB_i}(\overset{(-)}{i_R}, \overset{(-)}{i_{IB}})BS_i & \\
\text{credits (ii)} & \text{IB credits} & CR_{IB_i} = cr_{IB_i}(\overset{(+)}{i_R}, \overset{(+)}{i_{IB}}, \overset{(-)}{i_B})BS_i, & (6) \\
\text{assets (iii)} & \text{bonds} & B_i^D = b_i^D(\overset{(+)}{i_B}, \overset{(-)}{i_{IB}})BS_i &
\end{array}
$$

For a proof see appendix 6.2.

3 Financial markets

For simplicity we assume that asset markets are dominated by banks' financial activities. Thus, we assume that activities of the private sector are marginal and do not need to be explicitly modeled. We also assume that there are two representative banks, bank i and bank j, that are needed to describe all other banks and show symmetries. All other banks behave like these two explicitly described banks.

Central bank money market In this static model the CB money market is characterized by the CB's liquidity supply, which is demanded by the banking sector, i.e., our two representative banks i and j.

For simplicity, we reduce the CB's policy to its main instrument: the short-term refinancing rate (MRO). We assume that this rate is determined according to an inflation target; it uses the Taylor rule.

Thus, the CB's policy is to determine the CB refinancing rate, abbreviated to the reserve rate $i_R = const$. Thus, the market for CB money is $M_{CB_i} + M_{CB_j} = M_{CB}$. The full description of the CB money market can be summarized in the asset market equation system (7-i).

Interbank credit market We assume a perfect interbank credit market without rigidities, asymmetries or frictions.

The banking sector can be divided into two groups according to the bank's position in the interbank credit market. A bank can be either a lender or a borrower in the interbank credit market, which is randomly defined.

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[1]Lower-case letters indicate shares of the balance sheet, e.g., $m_{CB_i} = M_{CB_i}/BS_i$.

Each group is represented by one bank, where i stands for a banking group, which provides its liquidity surplus on the interbank credit market and acts as a lender (CR_{IB_i}), whereas j stands for the group with a liquidity deficit, which demands credit on the interbank market, and as such represents the borrowers in the interbank market $(CR^D_{IB_j})$.

We assume a random group allocation, which is rooted in a change in CB reserves. This reaction stems from the assumption that interbank credit supply can be substituted for (excess) CB reserves. Figure 2 shows the relation between the interest rate on interbank credits in the Euro area (EONIA) and excess reserve holdings at the CB. It displays a negative relationship between the interbank interest rate and the excess reserve holdings at the CB. Consequently, it could be assumed that in case of a lower yield on interbank lending, banks switch to the alternative asset of (excess) CB reserve holdings.
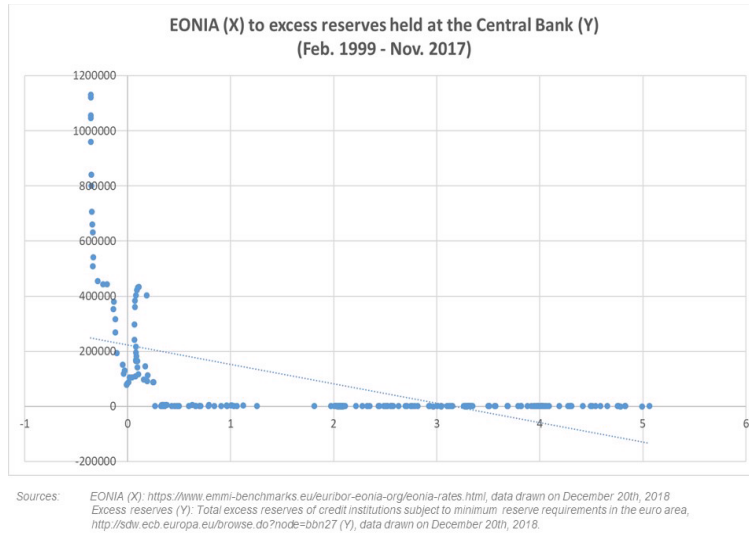

Figure 2: Interest rate in the interbank market in relation to excess reserves held at the CB

If the change in CB reserve holdings is positive banks i experience a liquidity surplus and face two investment opportunities, i.e. to act as lender in the interbank market $(CR_{IB_i} > 0)$ or to invest in the bond market $(B^D > 0)$. If the change in reserves is negative, banks j experience a liquidity shortage and face two refinancing opportunities, i.e., to borrow on the interbank market $(CR_{IB_j} < 0)$ or from the CB $(M_{CB_j} > 0)$. Bank j's demand in the interbank credit market is CR^D_{IBj}. This demand is matched by the credit supply of bank i (CR_{IB_i}). Consequently, a liability in the balance sheet of bank j, which is

matched by an asset of bank i, can be written as $L_{IB_j} = CR_{IB_i}$, which means that the balance sheet of the borrowing bank j records a "negative" credit supply $(-CR_{IBj}) = L_{IB_j} = CR_{IB_i}$.

In aggregate, the interbank credit market clears $(CR_{IB_i} - CR_{IBj}^D = 0)$. The full description of this market can be summarized in the asset market equation system (7-ii).

Bond market We assume a government bond (B) as a representative investment opportunity in all kinds of assets. Domestic bonds are demanded by all banks. The supply of the number of bonds is exogenous and the price is determined by market forces, $B_i^D + B_j^D = P_B B$. The full description of this market can be summarized in the asset market equation system (7-iii).

3.1 Complete financial market system and equilibrium

The complete system of financial markets can be summarized as

$$
\begin{array}{lll}
\text{market for} & & \\
\text{CB money (i)} & m_{CB_i}(\overset{(-)}{i_R},\overset{(-)}{i_{IB}})BS_i + m_{CB_j}(\overset{(-)}{i_R},\overset{(-)}{i_{IB}})BS_j = M_{CB} & \\
\text{IB credits (ii)} & cr_{IB_i}(\overset{(+)}{i_R},\overset{(+)}{i_{IB}},\overset{(-)}{i_B})BS_i = cr_{IB_j}^D(\overset{(+)}{i_R},\overset{(-)}{i_{IB}},\overset{(-)}{i_B})BS_j & (7) \\
\text{bonds (iii)} & b_i^D(\overset{(+)}{i_B},\overset{(-)}{i_{IB}})BS_i + b_j^D(\overset{(+)}{i_B},\overset{(-)}{i_{IB}})BS_j = P_B B &
\end{array}
$$

As we have three asset markets, we can determine three endogenous variables, namely i_{IB}, i_B, M_{CB} . Each of these variables depends on the vector of exogenous variables, namely (i_R, B).

Proposition 2 *Financial market system (7) implicitly defines a vector of equilibrium rates of return and asset prices, namely $\tilde{i}_{IB}, \tilde{i}_B, \tilde{M}_{CB}$. Each of these variables depends on the vector of exogenous variables (i_R, B); namely $\tilde{i}_{IB_i} = i_{IB_i}(i_R, B)$, $\tilde{i}_B = i_B(i_R, B)$ and $\tilde{M}_{CB} = M_{CB}(i_R, B)$.*
For a proof see appendix 6.2

This portfolio equilibrium was realized in the individual bank's balance sheet. Bank i takes individual portfolio decisions to realize this stock equilibrium in the end of the period. In the next section, we examine a dynamic flow mechanism, which will lead the bank into this portfolio equilibrium.

3.2 Flow mechanism in the interbank credit market

Several studies identify a credit boom as potential root cause of a crisis (see, e.g., Claessens, 2014, Dell'Arriccia et al., 2008). We acknowledge the importance of the interbank credit market and focus on the interbank credit market in aggregate (displayed in the financial market system in equation (ii)). Here,

interbank credit demand and supply are matched. The interbank credit market is a very short-term, overnight market, which induces a roll-over risk with respect to the portfolio management of the individual bank i. Departing from the notion that is generally assumed in the literature, in this context "roll-over risk" is not associated with a sole interest rate risk to the borrowing bank, but with a risk for the lender to have to adjust its portfolio management. In this model, the banking sector can be divided into two groups according to their position in the interbank credit market, where banks in group i supply their liquidity surplus and act as lenders, whereas banks of group j hold a liquidity deficit, demand credit on the interbank market, and as such represent borrowers in the interbank market. Therefore, the interbank credit supply of bank i is modelled in a periodic flow concept, which can guide the financial system into a stable or instable equilibrium. This dynamic adjustment process is examined below.

In the previous section we already determined the stationary equilibrium of the asset stock. Thus, at the end of all flow adjustments there must be a stationary stock equilibrium as described in section 3.1. Further, the flow process has to be consistent with the asset stock equilibrium, such that the flow process will eventually end in the stock equilibrium. Thus, we need to model the flow adjustment accordingly. We model an adjustment period and assume that stock adjustments are not instantaneously but rather take a few days until the flow process terminates in the new stock equilibrium.

In general, the change in the stock of interbank credits recorded in bank i's balance sheet's asset side (i.e. interbank credit supply) ($\dot{C}R_{IB_i}$) is determined by newly created or revolved credit contracts as well as by a dissolution of interbank credit provision . More precisely, $\dot{C}R_{IB}^g$ are newly created or revolved interbank credits that generate a gross increase in the stock of interbank credits, while dissolving existing interbank credits ($\dot{C}R_{IB_i}^\delta$) lead to a gross reduction in the stock of interbank credits of bank i. Consequently, the change in the stock of interbank credit provision recorded in bank i's balance sheet is broken down into the creation of new credits and reduction of existing interbank credits.

$$\dot{C}R_{IB_i}(t) = \dot{C}R_{IB_i}^g(t) + \dot{C}R_{IB_i}^\delta(t) \tag{8}$$

First, the creation of interbank credits, $\dot{C}R_{IB_i}^g$: With the notion of being on the way to the new equilibrium, the bank has an idea of its equilibrium credit demand $CR_{IB_i} = CR_{IB_j}^D(\overset{(+)}{i_R}, \overset{(-)}{i_{IB_i}}, \overset{(-)}{i_B}) = CR_{IB_j}^D(\overset{(+)}{i_R})$. The bank also knows the current level of its credit supply CR_{IB_i}. As long as credit demand exceeds credit supply at equilibrium, the bank provides more credit to the market. To speed up this adjustment process, the difference between the stock of credit demand $CR_{IB_i}^D$ and the already created supply CR_{IB_i} translates into a newly generated credit flow.

$$\dot{C}R_{IB_i}^{g1}(t) = b\left(CR_{IB_i}^D(i_R, i_{IB_i}) - CR_{IB_i}(t)\right),$$

where b is a parameter that translates excess demand in stocks into a credit-creating flow activity. Further, some of the existing stock of credit relations

with other banks can be easily used for a revolving mechanism. The decision of bank i to roll-over credit at similarly favorable conditions as for the last overnight credit does not take place automatically, but takes into account the targeted portfolio equilibrium and the respective interest rate in the interbank credit market (i_{IB_i})

$$\dot{C}R_{IB_i}^{g2}\left(t\right) = \rho(i_{IB_i})\left(CR_{IB_i}\left(t\right)\right)^{1-\alpha},$$

with $\alpha < 1$ and ρ as a parameter that refers to the traditional relationships to borrowing banks that develop over time (see e.g. Cocco et al., 2009; De la Motte et al., 2010; Afonso et al. 2013 on relationship lending).

Second, dissolving existing interbank credits, $\dot{C}R_{IB_i}^{\delta}$: While bank i is on its way to the new equilibrium stock of interbank credits, interbank lending is very short-term and consists of mostly overnight credits that are either renewed or not. Thus, at every point in time t (every day) a large number of these credits are repaid. Again, for simplicity we assume that all credits are overnight credits and paid back every day. As we assume no defaults in normal interbank relations, all of the existing stock CR_{IB_i} at time t is dissolved

$$\dot{C}R_{IB_i}^{\delta}\left(t\right) = -CR_{IB_i}\left(t\right).$$

Net credit dynamics Total credit dynamics can now be described by bringing the two components together:

$$\dot{C}R_{IB_i}\left(t\right) = b\left(CR_{IB_i}^{D}(i_R) - CR_{IB_i}\left(t\right)\right) + \rho(i_{IB_i})\left(CR_{IB_i}\left(t\right)\right)^{1-\alpha} - CR_{IB_i}\left(t\right).$$
$$(9)$$

Equation (9) is a non-linear, non-homogeneous differential equation in CR_{IB_i}. The equation

$$\dot{C}R_{IB_i}\left(t\right) = bCR_{IB_i}^{D} + \rho CR_{IB_i}^{1-\alpha} - \left(1+b\right)CR_{IB_i}$$

is graphically described in figure 3.and the properties of this differential equation are given by

$$\frac{d\dot{C}R_{IB_i}}{dCR_{IB_i}} = \rho\left(1-\alpha\right)CR_{IB_i}^{-\alpha} - \left(1+b\right)\begin{cases} > 0 \text{ for } CR_{IB_i}^{\alpha} > \rho\left(1-\alpha\right)\left(1+b\right)^{-1} \\ = 0 \text{ for } CR_{IB_i}^{\alpha} = \rho\left(1-\alpha\right)\left(1+b\right)^{-1} \\ < 0 \text{ for } CR_{IB_i}^{\alpha} < \rho\left(1-\alpha\right)\left(1+b\right)^{-1} \end{cases}$$

$$\frac{d\dot{C}R_{IB_i}^{2}}{d\left(CR_{IB_i}\right)^{2}} = \rho\left(-\alpha+\alpha^{2}\right)CR_{IB_i}^{-\alpha-1} < 0$$

A qualitative analysis of the dynamics of the process indicates that under the described standard conditions we have a stable dynamic process leading to the final stationary portfolio equilibrium (3.1). For this dynamic process we can also derive the stationary equilibrium at the end of the process when
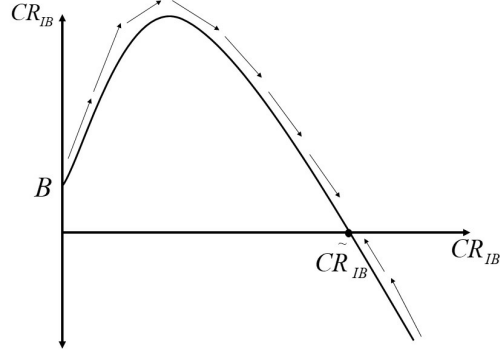
Figure 3: Dynamics and stability in the interbank credit market under normal conditions

$CR_{IB_i}^D(\overset{(+)}{i_R}, \overset{(-)}{\tilde{\imath}_{IB_i}}, \overset{(-)}{\tilde{\imath}_B}) - CR_{IB_i}(t) = 0$, and explicitly determine the steady state equilibrium credit supply[2]

$$CR_{IB_i} = \rho(\overset{(-)}{i_R})^{\frac{1}{\alpha}} \tag{10}$$

With (10) we have also shown that the portfolio equilibrium is consistently described.

Further, in order to provide credit, bank i has to have access to CB reserves. In other words, the adjustment towards a new portfolio equilibrium with a higher level of credit supply for bank i also means an adjustment of the portfolio equilibrium value of CB money M_{CB}. However, in this context we are more interested in the interbank credit flow mechanism. The flow management of reserves requires that bank i needs a reserve flow of $R(t) \geqq \dot{CR}_{IB_i}(t)$ to provide all credits described in the credit dynamics. In other words, as reserves are necessary for credit provision, the reserve flows $R(t)$ can be a direct constraint for credit expansion. Therefore, interbank credit managers determine the path of credit creation and plan the respective reserve flows for each point in time.

---

[2]

$$
\begin{aligned}
\dot{CR}_{IB_i}(t) &= 0 = bCR_{IB_i}^D + \rho(\overset{(-)}{i_R}, \overset{(+)}{i_{IB_i}})CR_{IB_i}^{1-\alpha} - (1+b)CR_{IB_i} \\
0 &= \rho(\overset{(-)}{i_R}, \overset{(+)}{i_{IB_i}})CR_{IB_i}^{1-\alpha} - CR_{IB_i} \\
1 &= \rho(\overset{(-)}{i_R}, \overset{(+)}{i_{IB_i}})CR_{IB_i}^{-\alpha} \\
CR_{IB_i} &= \rho(\overset{(-)}{i_R}, \overset{(+)}{i_{IB_i}})^{1/\alpha}
\end{aligned}
$$

13

However, as reserve in- and outflows are stochastic the planned reserves have to take this stochastic element into account. If x is a random in- or outflow of reserves, with $E[x] = 0$ and $R^p(t)$ the planned reserves, then according to the expected availability $E[x]$ and reserve requirements $R(t)$, credit managers would plan

$$R^p(t) = \dot{C}R_{IB_i}(t) - E[x].\qquad(11)$$

Managers have a consistent plan and therefore, in expected values $(E[x] = 0)$ the plan works, and figure 3 represents the dynamics provided expectations are fulfilled,

$$R^p(t) + E[x] = \dot{C}R^p_{IB_i}(t) = b\left(CR^D_{IB_i} - CR_{IB_i}(t)\right) + \rho\left(CR_{IB_i}(t)\right)^{1-\alpha} - (1+b)CR_{IB_i}(t).$$

However, real conditions are sometimes different than the expected values. Therefore, we now describe what happens to the adjustment process if the bank randomly realizes reserve values other than those expected. From the above discussion we know that existing stochastic reserves can restrict the credit creation process $R(t) \geqq \dot{C}R_{IB_i}(t)$. Therefore, the manager has planned reserves $R^p(t)$, expecting that the stochastic element of resource flows is zero $(E[x] = 0)$. However, $x = R^R$ is stochastic and therefore in reality x may randomly realize the value $R^R < 0$ within the period of adjustment. Then, $R(t) = R^p(t) + R^R$ and real credit creation is restricted to $\dot{C}R_{IB_i}(t) = R^p(t) + R^R$. As the planned credit creation at that point in time is $\dot{C}R^p_{IB_i}(t)$, the dynamics of the adjustment process fall short of this new reality. Adjusting to this real world observation $R(t) = R^p(t) + R^R < R^p(t)$, the bank will have to switch to a new adjustment path because of the realized reserve constraint. The new adjustment path is now[3]

$$\dot{C}R_{IB_i}(t) = R^R + bCR^D_{IB_i} + \rho'CR^{1-\alpha}_{IB_i}(t) - (1+b)CR_{IB_i}(t)$$

As long as the random shock $R^R < 0$ is sufficiently small in absolute terms $|R^R|$ the intersection with the vertical axis in figure (3) remains positive, and we have no general change in the adjustment dynamics. However, if in absolute terms the random shock $R^R < 0$ is sufficiently large $B = R^R + bCR^D_{IB_i}$ may turn negative, and the properties of the dynamic adjustment process may change. Figure 4 shows this new path for a negative B. Before we can discuss the implications for the dynamics we need to formally identify the new, low steady state and the reactions of this steady state with respect to changes in variables. We determine the low steady state for $CR^D_{IB_i} - CR_{IB_i} > 0$

---

[3]

$$\dot{C}R_{IB_i}(t) = R(t) = R^p(t) + R^R$$
$$\dot{C}R_{IB_i}(t) = R^R + b\left(CR^D_{IB_i} - CR_{IB_i}(t)\right) + \rho\left(CR_{IB_i}(t)\right)^{1-\alpha} - (1+b)CR_{IB_i}(t)$$
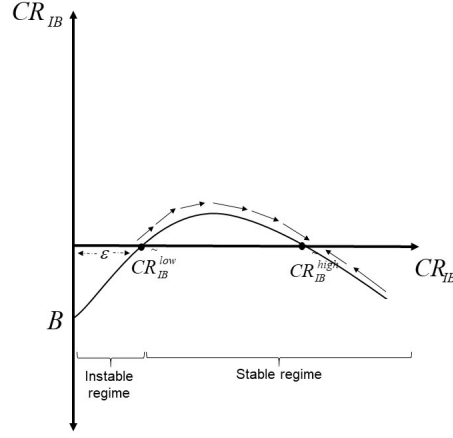
Figure 4: Dynamics and instability in the interbank market

$$0 = G = R^R + bCR_{IB_i}^D + \rho' CR_{IB_i}^{1-\alpha} - (1+b)\, CR_{IB_i} \qquad (12)$$

As this equation cannot be solved explicitly, we need to apply the implicit function theorem. Looking at figure 4 and using the implicit function theorem at a local point, we can state that equation (12) implicitly defines a function CR_{IB_i} at potentially two equilibrium points, a low equilibrium point $CR_{IB_i}^{low}$ and a high equilibrium point $CR_{IB_i}^{high}$[4]

$$CR_{IB_i}^{low} = CR_{IB_i}^{low}(R^R, ...), \qquad \text{with} \quad \frac{CR_{IB_i}^{low}}{dR^R} > 0 \qquad (13)$$
$$CR_{IB_i}^{high} = CR_{IB_i}^{high}(R^R, ...),$$

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[4]Implicit function theorem: $0 = G = R^R + bCR_{IB_i}^D + \rho' CR_{IB_i}^{1-\alpha} - (1+b)\, CR_{IB_i}$ implicitly defines a function $CR_{IB_i}^{low} = CR_{IB_i}^{low}(R^R, ...)$ if $\frac{dG}{dCR_{IB_i}} \neq 0$: Thus, taking the derivative of G with respect to CR_{IB_i} gives

$$\frac{\partial G}{\partial CR_{IB_i}} = (1-\alpha)\,\rho' CR_{IB_i}^{-\alpha} - (1+b) > 0 \qquad \text{for} \quad CR_{IB_i} < \left(\frac{\rho'(1-\alpha)}{1+b}\right)^{\frac{1}{\alpha}}$$

$$= (1-\alpha)\,\rho' CR_{IB_i}^{-\alpha} - (1+b) < 0 \qquad \text{for} \quad CR_{IB_i} > \left(\frac{\rho'(1-\alpha)}{1+b}\right)^{\frac{1}{\alpha}}.$$

Thus, $CR_{IB_i}^{low} = CR_{IB_i} < \left(\frac{\rho'(1-\alpha)}{1+b}\right)^{\frac{1}{\alpha}}$ is the low credit level stationary equilibrium. The derivative of this implicit function is $\frac{\partial G}{\partial R^R} = 1$

$$\frac{dCR_{IB_i}}{dR^R} = -\frac{\frac{\partial G}{\partial R^R}}{\frac{\partial G}{\partial CR_{IB_i}}} = -\frac{1}{\frac{\partial G}{\partial CR_{IB_i}}} < 0$$

.

15

In figure 4 we use this new path for a qualitative dynamic analysis. While figure 3 described a global overall stable process with only one equilibrium, figure 4 shows two equilibria. In this figure the high equilibrium is comparable to the one equilibrium in figure 3. We see a locally stable process as the $\dot{C}R_{IB_i}(t)$-*curve* has a negative slope around the high equilibrium. The high equilibrium is a locally stable point. This is different for the low equilibrium. Here, the slope is positive, which implies that the low equilibrium $CR_{IB_i}^{low}$ is locally unstable. As a result we have two dynamic regimes. At points larger than the low equilibrium level $CR_{IB_i}^{low}$ the credit creation process will be on a stable path and move to the high equilibrium $CR_{IB_i}^{high}$, which is also the final portfolio equilibrium the banks would like to reach. However, if we look at points below the low equilibrium level $CR_{IB_i}^{low}$ the process is unstable and bank i would keep on decreasing credit creation. In this case, the credit creation of bank i is constrained by too low a reserve inflow $R(t)$ and may reduce to zero. This brief discussion already indicates that adjustment processes are no longer only stable. If the stochastic shock described by a randomly much lower reserve inflow in the adjustment process is sufficiently large or CR_{IB_i} is still rather low, such that $CR_{IB_i}(t) < CR_{IB_i}^{low}$, the process becomes unstable. This critical mechanism is studied in more detail below.

3.3 Resilience of interbank credit creation adjustments

Knowing the unstable credit creation processes in a bank that provides interbank credits, we now discuss the aggregate process in the interbank market as well as some elements that potentially affect the resilience of this market. The term *resilience* in this context stands for the likelihood of the market to be in a stable regime and automatically returning to the stable portfolio equilibrium.

As the realized random reserve flow $x = R^R$ constrains the actual credit creation activity $(R(t) \geqq \dot{C}R_{IB_i}(t))$, and by that determines in (14) the shape of the actual adjustment process

$$d\dot{C}R_{IB_i}(t) = R^R + bCR_{IB_i}^D + \rho' CR_{IB_i}^{1-\alpha} - (1+b)CR_{IB_i}, \qquad (14)$$

these random reserve flows must be studied in more detail.

While x is a particular realized value during the period of adjustment, we know more about the random distribution of this shock and can use this knowledge to describe the likelihood of the process remaining stable at any moment during the adjustment period.

First, as described before x is a random in- or outflow of reserves, with expectation $E[x] = 0$ and $Var = \sigma^2$. We specify the random distribution by choosing a normal distribution

$$X \sim \mathcal{N}(0, \sigma^2) \quad \text{with} \quad f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}. \qquad (15)$$

Thus, we may be able to determine probabilities for each value x of the random in- or outflow of reserves during the adjustment period. However, we need to

find out more about the probability of instability.

Second, according to figure 4 a process becomes unstable if the low equilibrium $CR_{IB_i}^{low}$ is to the right of the current credit stock $CR_{IB_i}(t)$ such that $CR_{IB_i}(t) \leq CR_{IB_i}^{low}$. As analyzed above, $CR_{IB_i}^{low}$ is determined by the realized reserve flow x. From (13) we know that for a particular value x the derivative of the low equilibrium with respect to x is $\frac{CR_{IB_i}^{low}}{dx} > 0$. Using a local and linear approximation at $CR_{IB_i}^{low}$ we can rewrite $CR_{IB_i}^{low}$ as the linear function

$$CR_{IB_i}^{low}(x) = g(x) = mx - c \tag{16}$$

With this linear approximation and the random distribution (15) we now arrive at the following proposition[5]:

Proposition 3 *(probability of instability) Using the approximation in figure 4 for the low equilibrium $CR_{IB_i}^{low}(x)$ and random distribution (17) for the flow of random reserves, we can derive the probability that ε is smaller than or equal to $CR_{IB_i}^{low}$ ($\varepsilon \leq CR_{IB_i}^{low}$), and thus that the adjustment process becomes unstable*

$$\mathbb{P}(f(X) \geq \varepsilon) = \int_{\varepsilon}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\left(\frac{1}{m}(x-c)\right)^2}{2\sigma^2}} \frac{1}{|m|} dx. \tag{17}$$

Therefore, if the current position of credit creation in figure 4 is $CR_{IB_i}(t) = \varepsilon$, we can now state a probability that the low equilibrium is to the right of ε, and thus state a probability of randomly falling in the unstable region of the adjustment process. In other words, we determine the probability of an unstable adjustment process.

If bank i is a representative credit provider in our system, all described mechanisms hold for the entire interbank credit market. Therefore, it is interesting to identify elements that can increase or decrease the probability of market instability, and identify the elements that affect *market resilience* (defined as the probability of market stability).

First, if ε increases, the probability of instability decreases

$$\frac{d}{d\varepsilon}\mathbb{P}(f(X) \geq \varepsilon) = \frac{d}{d\varepsilon}\int_{\varepsilon}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\left(\frac{1}{m}(x-c)\right)^2}{2\sigma^2}} \frac{1}{|m|} dx = -\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\left(\frac{1}{m}(\varepsilon-c)\right)^2}{2\sigma^2}} \frac{1}{|m|} < 0. \tag{18}$$

---

[5]

$$\begin{aligned} \mathbb{P}(f(X) \geq \varepsilon) &= \int_{x \leq \varepsilon} f_X(g^{-1}(x)) \frac{1}{|g'(g^{-1}(x))|} dx \\ &= \int_{x \geq \varepsilon} f_X\left(\frac{1}{m}(x-c)\right) \frac{1}{|m|} dx \\ &= \int_{\varepsilon}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\left(\frac{1}{m}(x-c)\right)^2}{2\sigma^2}} \frac{1}{|m|} dx \end{aligned}$$

That is, close to the high equilibrium the probability of instability is low. The market is rather resilient.

Second, it is interesting to note that it is not necessarily the level of the reserve flows that determines the probability of falling into an unstable region of the adjustment process. If we take the derivative of (17) with respect to the variance σ^2, the probability of instability increases

$$\frac{d}{d\sigma^2}\mathbb{P}(f(X) \geq \varepsilon) = \frac{1}{|m|}\frac{1}{\sqrt{2\pi\sigma^2}}\frac{\varepsilon}{2\sigma^2}e^{-\frac{\varepsilon^2}{2m^2\sigma^2}} > 0. \tag{19}$$

Thus, high volatility of reserve flows, which may be generated even in other financial markets,[6] affects stability and resilience of the interbank credit market. When market volatility increases, shocks to individual banks in the group i are assumed to be correlated and thus, non-diversifiable, which implies that lenders in the interbank credit market are affected simultaneously. The interbank credit market may fall into an unstable adjustment mechanism with more volatile reserve flows. That is, the interbank market is more likely to become unstable if shock or developments somewhere in the financial system cause higher volatility of reserve flows.

4 Model implications

The theoretical considerations of the model presented above add to the general understanding of the mechanisms and dynamics in the interbank market. The results show that dynamics in the interbank credit market can guide the financial system towards an unstable equilibrium and lead to systemic risk. In the extreme case, when banks are stuck in the low equilibrium loan provision in the interbank credit market could collapse entirely with the interbank credit market drying up. When this threat is present the CB faces a trade-off between rescuing a bank, which involves simultaneously a moral hazard issue or the bank becoming insolvent, which could have contagious effects in the interbank market. In 2008, the ECB decided to rescue individual banks and thereby the financial system as the whole. The Central Bank prevented the realization of systemic risk, at the cost of a moral hazard incentive in the banking sector (see, e.g., Farhi and Tirole, 2012 on moral hazard and systemic bailouts). Monetary policy can try to revitalize the interbank credit market with the help of additional emergency liquidity supply at the cheapest refinancing rates.

To safeguard liquidity within the interbank credit market, either the CB has to meet excess demand for credit in the interbank market, or the shock on reserves has to be reduced by preventing volatility and bubbles. Therefore, macroprudential policies, risk indicators of triggers, and amplifiers of volatility and early warning systems are important so that countermeasures can be taken.

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[6]Reasons for higher volatility in other financial markets are shown, for instance, by Daniel et al. (1998). Their model shows that investor overconfidence can cause stock market bubbles and increase volatility in asset markets.
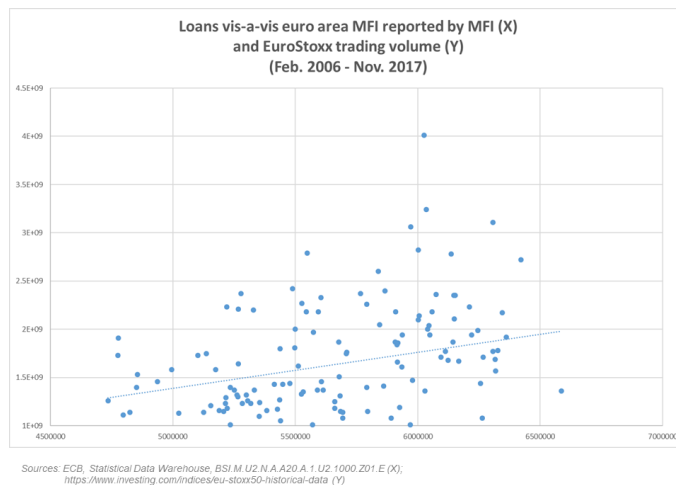
Figure 5: Relation between stock of IB credits and the trade volume of the EuroStoxx

Volatility in asset markets We assume that the volatility in asset markets triggers these dynamics and as such is strongly connected to the liquidity management of a given bank. The theoretical argument has been proven by, e.g., Daniel et al. (1998) who show that investor overconfidence can cause increased volatility in returns and asset markets, which is subsequently reflected in the bank's balance sheet and portfolio management. In the same vein, Chuang and Lee (2006) examine the overconfidence hypothesis in an empirical framework and find some evidence for it. In order to be more precise with respect to our model, we conduct a first check on the relationship between tvolatility on other markets and the interbank credit market with the help of empirical data. Data available as indicators of interbank loans are drawn from the ECB's dataset "Balance sheet items," which refers to the aggregated balance sheet of Euro area MFIs. We take "Loans vis-a-vis euro area MFI reported by MFI excluding ESCB in the euro area (stock)" as a proxy for our purposes. In figure 5 we show the relationship between interbank credits and the trade volume of EuroStoxx shares. Looking at this diagram, the trade volume in the stock market seems to be connected with the interbank credit market ($corr$(IB credits, trading volume) $= 0.290$) . That is, active stock market trading is connected to an increase in interbank credit activities. The interbank credit market is related to balancing stock market trading activities. The relationship between trade volume and asset price volatility is described in figure 6 ($corr$(trading volume, volatility index) $= 0.511$) Those two positive correlations suggest a positive correlation between the interbank credit provision and volatility, which should, however, be further explored and proved.

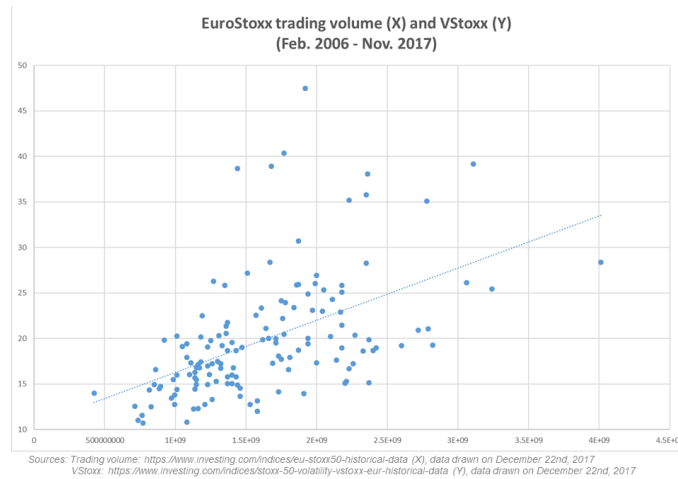EuroStoxx trading volume (X) and VStoxx (Y)
(Feb. 2006 - Nov. 2017)

Figure 6: Relation of trade volume of the EuroStoxx and volatility index of the EuroStoxx

Furthermore, volatility in banks' balance sheet management could be changed by monetary policy. The monetary policy regime could have an asymmetric impact on the resilience of the interbank credit market. By tightening up monetary policy, the CB decreases the money supply or increases the monetary policy rate. The transmission channels of monetary policy suggest a mechanism working through a reduction in spending on the capital markets, which decreases asset prices (known as an "asset price channel"). The reduction in asset prices means also higher asset price volatility. This positive correlation between the CB's refinancing rate and asset price volatility appears to be reflected in figure 1 above. Overall, a positive correlation could be assumed to hold between the monetary policy rate and asset price volatility. If this is true a contractionary monetary policy would go hand in hand with higher asset price volatility. This would also increase the volatility in reserve flows in banks' balance sheet management, too. Simultaneously, the rise in interest rates implies that banks' can generate higher yields through interbank lending, which increases their willingness to supply credit to interbank markets. We suggest that banks holding a liquidity surplus substitute interbank lending for holding (excess) reserves at the CB in their portfolio management. Consequently, a rise in policy rates would increase the credit supply in interbank markets. In our model, contractionary monetary policy would increase the probability of the interbank market entering an unstable regime. When the volatility of reserve flows increases, the probability of an unstable adjustment process in the interbank market increases and thus interbank market resilience decreases.

By contrast, an expansionary monetary policy means an increase in the

money supply and a decreases in the policy rate, respectively. Following the same line of argumentation, expansionary policy would lower asset price volatility and hence add to interbank credit market resilience as it reduces the probability of the interbank market entering an unstable regime. Recently, CB policymakers have considered raising policy rates again and introducing a contractionary monetary policy regime. When switching the regime, policymakers should be aware of a potential reduction in interbank credit market resilience and the consequences for financial stability.

Macroprudential policies   Macroprudential policies were developed to mitigate systemic risk. Several tools (e.g., countercyclical capital requirements, Liquidity Coverage Ratio (LCR)) have been implemented to ensure the liquidity of a bank at all times. In the context of our model, they should also safeguard the ability of a bank to provide interbank credit at all times, whereby a limited cash inflow constrains its credit provision. However, the discussion on the effectiveness of these tools is still underway.

Further tools measure each bank's contribution to systemic risk within the concept of "Systemic Expected Shortfall" (see Archaya et al., 2010) or assess interconnectedness across banks and systemic risk at the bank level, e.g. with respect to bank size, loan growth, leverage, or loan maturity (the concept of $\Delta CoVar$). However, while these measures monitor an individual bank's contribution to systemic risk, they do not focus on the risk inherent in financial markets, namely the interbank credit market but also other financial markets which can transfer risk via volatility.

In our view, the risk of asset volatility with respect to its effects on the bank's portfolio and liquidity management, is still underrepresented in macroprudential policies and should be given further attention.

5   Conclusion

The lack of theoretical studies on systemic risk has induced us to develop a theoretical model of systemic risk factors in the financial system with a focus on interbank credit markets. Furthermore, we pay special attention to the interbank credit market and its dynamic adjustment processes without losing track of the individual bank's portfolio management. Starting with a single bank's balance sheet, which includes interbank activities, we derive general portfolio equilibria in financial markets. Based on these static equilibria, a stochastic model of dynamics in the interbank market is introduced, which adds with a dynamic view on sources of systemic risk. This source is a potential unstable equilibrium in the interbank credit market, which can be realized due to a stochastic process that defines CB money available for interbank credit provision. Defining the probability of interbank market stability as market resilience, the volatility of reserve flows may threaten the resilience of interbank markets and in turn of the entire financial system.

The resilience of the financial system may improve when liquidity in the interbank market is monitored, e.g., with the help of indicators, such as the liquidity coverage ratio of macroprudential policies. However, the risk is rooted in the volatility of reserve flows, which is caused by stochastic volatility shocks in other markets. The CB can attempt to stabilize the interbank credit market or substitute interbank credit flows with the help of expansionary monetary policy. However, it is important to note that monetary policy could incidentally also reduce financial stability. We identify a potential risk to financial stability stemming from a monetary policy regime-switch from an expansionary to a contractionary policy, which has a particular prominence in the recent discussion of monetary policy tightening. We stress that contractionary monetary policy can lead to a higher probability of an unstable adjustment process in interbank markets and a decrease in financial market resilience.

Consequently, we emphasize the importance of system-wide, macroprudential policies that should pay special attention to the risk of asset price volatility and its effects on bank's portfolio and liquidity management to support and ensure the resilience of the financial system. Moreover, policymakers should be aware of asymmetric effects of different policy regimes on financial market resilience.

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6 Appendix

6.1 A1 Proof of proposition 1: Asset demand function derived from portfolio choice model.

Problem

$$\pi_i = -i_R M_{CB_i} + i_{IB} CR_{IB_i} + i_B B_i^D$$

$$\max \quad : \quad V_i = V_i\left(\pi_i, \sigma_i\left(M_{CB_i}, CR_{IB_i}, B_i^D\right)\right)$$
$$s.t. \quad : \quad M_{CB_i} + CR_{IB_i} + P_B B_i^D = BS_i$$

$$\mathcal{L} \quad = \quad V_i\left(\pi_i, \sigma\left(M_{CB_i}, CR_{IB_i}, B_i^D\right)\right)$$
$$-\lambda\left(M_{CB_i} + CR_{IB_i} + P_B B_i^D - BS_i\right)$$

This yields the following first order conditions of our optimization problem.

F.O.C.

$$(i) \quad A_1 \;=\; \frac{d\mathcal{L}_i}{dM_{CB_i}} = -i_R + \frac{\partial V_i}{\partial \sigma_i}\frac{\partial \sigma_i}{\partial M_{CB_i}} - \lambda = 0,$$

$$\frac{d_i^2\mathcal{L}}{d\left(M_{CB_i}\right)^2} = A_{11} = \frac{\partial V_i}{\partial \sigma_i}\overset{(-)}{\frac{\partial^2 \sigma_i}{\partial(M_{CB_i})^2}} < 0, \qquad \frac{\partial^2 \sigma_i}{\partial(M_{CB_i})^2} > 0$$

$$(ii) \quad A_2 \;=\; \frac{d\mathcal{L}_i}{dCR_{IB_i}} = i_{IB} + \frac{\partial V_i}{\partial \sigma_i}\frac{\partial \sigma_i}{\partial CR_{IB_i}} - \lambda = 0, \quad \frac{d^2\mathcal{L}_i}{d\left(CR_{IB_i}\right)^2} = A_{22} = \frac{\partial V_i}{\partial \sigma_i}\overset{(+)}{\frac{\partial^2 \sigma_i}{\partial\left(CR_{IB_i}\right)^2}} < 0$$

$$(iii) \quad A_3 \;=\; \frac{d\mathcal{L}_i}{dB_i^D} = i_B + \frac{\partial V_i}{\partial \sigma_i}\frac{\partial \sigma_i}{\partial B_i^D} - \lambda = 0, \qquad \frac{d^2\mathcal{L}_i}{d\left(B_i^D\right)^2} = A_{33} = \frac{\partial V_i}{\partial \sigma_i}\overset{(+)}{\frac{\partial^2 \sigma_i}{\partial\left(B_i^D\right)^2}} < 0$$

$$(iv) \quad A_4 \;=\; \frac{d\mathcal{L}_i}{d\lambda} = M_{CB_i} + CR_{IB_i} + P_B B_i - BS_i = 0$$

This results in the following Jacobian matrix:

$$\begin{pmatrix} \overset{(-)}{\frac{\partial V_i}{\partial \sigma_i}\frac{\partial^2 \sigma_i}{\partial(M_{CB_i})^2}} & 0 & 0 & -1 \\ 0 & \overset{(+)}{\frac{\partial V_i}{\partial \sigma_i}\frac{\partial^2 \sigma_i}{\partial\left(CR_{IB_i}\right)^2}} & 0 & -1 \\ 0 & 0 & \overset{(+)}{\frac{\partial V_i}{\partial \sigma_i}\frac{\partial^2 \sigma_i}{\partial\left(B_i^D\right)^2}} & -1 \\ 1 & 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} \overset{(-)}{A_{11}} & 0 & 0 & -1 \\ 0 & \overset{(-)}{A_{22}} & 0 & -1 \\ 0 & 0 & \overset{(-)}{A_{33}} & -1 \\ 1 & 1 & 1 & 0 \end{pmatrix} = J$$

The equation system can be rewritten in the following matrix notation.

$$Jx = b$$

$$\begin{pmatrix} \overset{(-)}{A_{11}} & 0 & 0 & -1 \\ 0 & \overset{(-)}{A_{22}} & 0 & -1 \\ 0 & 0 & \overset{(-)}{A_{33}} & -1 \\ 1 & 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} dM_{CB_i} \\ dCR_{IB_i} \\ dB_i^D \\ d\lambda \end{pmatrix} = \begin{pmatrix} di_R \\ -di_{IB} \\ -di_B \\ dBS_i \end{pmatrix}$$

Implicit function theorem:

To show that the system has a solution, we apply the implicit function theorem. First, show that $|J| \neq 0$ and calculate the determinant of J with Laplace's formula.

$$|J| = \begin{vmatrix} \overset{(-)}{\mathbf{A}_{11}} & \mathbf{0} & \mathbf{0} & -\mathbf{1} \\ 0 & \overset{(-)}{A_{22}} & 0 & -1 \\ 0 & 0 & \overset{(-)}{A_{33}} & -1 \\ 1 & 1 & 1 & 0 \end{vmatrix}$$

$$= \mathbf{A}_{11}(-1)^{(1+1)} \begin{vmatrix} A_{22} & 0 & -1 \\ 0 & A_{33} & -1 \\ 1 & 1 & 0 \end{vmatrix} + (-\mathbf{1})(-1)^{(1+4)} \begin{vmatrix} 0 & A_{22} & 0 \\ 0 & 0 & A_{33} \\ 1 & 1 & 1 \end{vmatrix}$$

$$= A_{11}(A_{33} + A_{22}) + 1 \begin{vmatrix} 0 & A_{22} & 0 \\ 0 & 0 & A_{33} \\ 1 & 1 & 1 \end{vmatrix}$$

$$= A_{11}A_{33} + A_{11}A_{22} + A_{22}A_{33}$$

$$|J| = \overset{(-)}{A_{11}}\overset{(-)}{A_{33}} + \overset{(-)}{A_{11}}\overset{(-)}{A_{22}} + \overset{(-)}{A_{22}}\overset{(-)}{A_{33}} > 0 \tag{20}$$

Portfolio adjustments Portfolio adjustments due to changes in exogenous variables (i_R, B) can be derived in the usual way. However, we do not go through this process for every single variable. We generally assume that direct (own) effects dominate cross effects and thus that the scheme of reactions should be in line with standard reactions.

Exemplary, we analyze the effect of a change in di_R (Δ) on dM_{CB_i}. The application of Cramer's rule yields $x_1 = dM_{CB_i} = \frac{|J_1|}{|J|}$. We use the first column to calculate $|J_1|$ with the help of Laplace's formula.

$$|J_1| = \begin{vmatrix} di_R & 0 & 0 & -1 \\ -di_{IB} & \overset{(-)}{A_{22}} & 0 & -1 \\ -di_B & 0 & \overset{(-)}{A_{33}} & -1 \\ dBS_i & 1 & 1 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} \mathbf{\Delta} & 0 & 0 & -\mathbf{1} \\ \mathbf{0} & \overset{(-)}{A_{22}} & 0 & -1 \\ \mathbf{0} & 0 & \overset{(-)}{A_{33}} & -1 \\ \mathbf{0} & 1 & 1 & 0 \end{vmatrix}$$

27

$$= \mathbf{\Delta}(-1)^{(1+1)} \begin{vmatrix} A_{22} & 0 & -1 \\ 0 & A_{33} & -1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= \Delta(A_{33})$$

$$|J_1| = \left[\Delta \overset{(-)}{A_{33}} \right], \qquad \Delta = 1$$

$$|J_1| = \overset{(-)}{A_{33}} < 0 \tag{21}$$

$$dM_{CB_i} = \frac{|J_1|}{|J|} < 0$$

In the same way we can calculate all other reactions. However, to save space we do not show all these calculations in this appendix. They may be obtained on request.

6.2   A2 Financial Markets and Equilibrium

Proof of proposition 2: Equilibrium price vector, derived using the implicit function theorem:

From the market system (7) we obtain a system of three functions F_0, F_1, F_2 depending on the three endogenous variables i_{IB}, i_B and M_{CB}. If these equations are linearly independent we can apply the implicit function theorem.

$$\text{CB money (i)} \quad F_0 = m_{CB_i}(\overset{(-)}{i_R}, \overset{(-)}{i_{IB}})BS_i + m_{CB_j}(\overset{(-)}{i_R}, \overset{(-)}{i_{IB}})BS_j - M_{CB} = 0$$
$$\text{IB credits } i \text{ (ii)} \quad F_1 = cr_{IB_i}(\overset{(+)}{i_R}, \overset{(+)}{i_{IB}}, \overset{(-)}{i_B})BS_i - cr^D_{IB_j}(\overset{(+)}{i_R}, \overset{(-)}{i_{IB}}, \overset{(-)}{i_B})BS_j = 0$$
$$\text{bonds (iii)} \quad F_2 = b^D_i(\overset{(+)}{i_B}, \overset{(-)}{i_{IB}})BS_i + b^D_j(\overset{(+)}{i_B}, \overset{(-)}{i_{IB}})BS_j - P_B B = 0$$

$$(22)$$

A detailed look at the CB money market shows, that this market is recursively related to the system and only required for determining the equilibrating supply of CB reserves. We reduce the system to (ii) and (iii). This results in the following Jacobian matrix:

$$J = \begin{pmatrix} \frac{\partial F_1}{\partial i_{LB}} & \frac{\partial F_1}{\partial i_B} \\ \frac{\partial F_2}{\partial i_{IB}} & \frac{\partial F_2}{\partial i_B} \end{pmatrix} = \begin{pmatrix} \overset{(+)}{\frac{\partial CR_{IBi}}{\partial i_{IB}}} - \overset{(-)}{\frac{\partial CR^D_{IBj}}{\partial i_{IB}}} & \overset{(-)}{\frac{\partial CR_{IBi}}{\partial i_B}} - \overset{(-)}{\frac{\partial CR^D_{IBi}}{\partial i_B}} \\ \overset{(-)}{\frac{\partial B^D_i}{\partial i_{IB}}} + \overset{(-)}{\frac{\partial B^D_j}{\partial i_{IB}}} & \overset{(+)}{\frac{\partial B^D_i}{\partial i_B}} + \overset{(+)}{\frac{\partial B^D_j}{\partial i_B}} \end{pmatrix} = \begin{pmatrix} \overset{(+)}{F_{11}} & \overset{(-)}{F_{12}} \\ \overset{(-)}{F_{21}} & \overset{(+)}{F_{22}} \end{pmatrix}$$

Here the numbering is defined as the first subindex giving the number of the market within equation system 22 and the second subindex gives the number

of the endogenous variable according to following definition $1 : i_{IB}$, $2 : i_B$. We assume that a change in the interest rate on bonds has a stronger effect on interbank credit demand than on interbank credit supply ($F_{12} < 0$).

Then, the reduced equation system can be rewritten in the following matrix notation:

$$Jx = b$$

$$
\begin{pmatrix}
\overset{(+)}{\dfrac{\partial CR_{IBi}}{\partial i_{IB}}} - \overset{(-)}{\dfrac{\partial CR_{IBj}^{D}}{\partial i_{IB}}} & \overset{(-)}{\dfrac{\partial CR_{IBi}}{\partial i_{B}}} - \overset{(-)}{\dfrac{\partial CR_{IBj}^{D}}{\partial i_{B}}} \\
\overset{(-)}{\dfrac{\partial B_{i}^{D}}{\partial i_{IB}}} + \overset{(-)}{\dfrac{\partial B_{j}^{D}}{\partial i_{IB}}} & \overset{(+)}{\dfrac{\partial B_{i}^{D}}{\partial i_{B}}} + \overset{(+)}{\dfrac{\partial B_{j}^{D}}{\partial i_{B}}}
\end{pmatrix}
\begin{pmatrix} di_{IB} \\ di_B \end{pmatrix}
=
\begin{pmatrix}
\overset{(+)}{\dfrac{\partial CR_{IBi}}{\partial i_{IB}}} - \overset{(+)}{\dfrac{\partial CR_{IBj}^{D}}{\partial i_{IB}}} \\
0
\end{pmatrix}
$$

$$
\begin{pmatrix}
\overset{(+)}{F_{11}} & \overset{(-)}{F_{12}} \\
\overset{(-)}{F_{21}} & \overset{(+)}{F_{22}}
\end{pmatrix}
\begin{pmatrix} di_{IB} \\ di_B \end{pmatrix}
=
\begin{pmatrix} \overset{(-)}{F_{1\Delta}} di_R \\ F_{2\Delta} di_R \end{pmatrix}
=
\begin{pmatrix} \overset{(-)}{F_{1\Delta}} di_R \\ 0 \end{pmatrix}
$$

Here $F_{1\Delta}, F_{2\Delta}$ is the derivative of F_1 and respectively of F_2 with respect to the exogenous variable di_R.

To apply the implicit function theorem $|J| \neq 0$:

Proof.

$$
|J| = \begin{vmatrix}
\overset{(+)}{F_{11}} & \overset{(-)}{F_{12}} \\
\overset{(-)}{F_{21}} & \overset{(+)}{F_{22}}
\end{vmatrix} = \overset{(+)}{F_{11}}\overset{(+)}{F_{22}} - \overset{(-)}{F_{21}}\overset{(-)}{F_{12}}
$$

The sign turns positive, if we assume that direct effects are in general large in absolute values:

$$
|J| = \overset{(+)}{F_{11}}\overset{(+)}{F_{22}} - (\overset{(-)}{F_{21}}\overset{(-)}{F_{12}}) > 0.
$$

■