Cost of credit, entrepreneurship, and misallocation

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Abstract

So far, the literature on financial constraints and misallocation has focused on a particular class of financial constraints, which prevents entrepreneurs from borrowing more than a fraction of their existing assets. Regardless of whether the constraint is binding or not, firms pay a constant, risk-free interest rate on the amount they effectively borrow, which might allow them to quickly accumulate internal funds and finance their investment without the collateral constraint being binding.

In this paper, I present some data that suggests that the heterogeneity in borrowing costs might actually be quite important, and that interest rates paid on debt correlate negatively with firm size, and positively with the ratio of debt to assets. I present and simulate a model that includes occupational choices and endogenously determined costs of credit, which could match some facts observed in the data. The main prediction of the model is that initial assets matter when it comes to firm selection, and that entrepreneurs which have not accumulated enough savings in the past face higher borrowing costs, which might significantly hinder the wealth accumulation process and force them to either get out of the market, or remain small.

I conduct numerical exercises and show that financial frictions can account for TFP losses of about 10% in France. my model also allows to estimate the impact of different policies designed to facilitate firms' access to credit. I show that credit subsidies and government guarantees on loans only have a moderate positive impact of TFP, since public authorities do not have better technologies to observe actual productivity and cannot discriminate productive entrepreneurs better than the private sector.

Keywords: Financial constraints, cost of credit, entrepreneurship, factor misallocation
JEL Classification: E44, G32, O33, O47

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1 Introduction

1.1 Related literature

A growing body of literature has recently focused on explaining cross-country variations in GDP per capita, not only by differences in technology or institutions, but also by arguing that differences in aggregate total factor productivity (TFP) are the result of a misallocation of factors of production between heterogeneous firms. In a seminal paper, Hsieh and Klenow (2009) [1] document a large heterogeneity in the measured returns on labor and capital across firms in India and China and argue that, if the factors of production were to be reallocated efficiently, to equalize the returns, aggregate TFP gains would be substantial in both countries (around 50% and 40%, respectfully).

More work has linked the dispersion of factor returns to the existence of financial constraints. In most of these papers, entrepreneurs finance expenditures by borrowing from financial intermediaries, at some interest rate, but are restricted in the amount they can borrow by collateral constraints. In Midrigan and Xu (2010) [2] for example, entrepreneurs can only borrow up to a fraction of their assets deposited at the bank, so that only a fraction of their expenses can be financed through debt, the rest being financed by savings or retained earnings. In a calibrated simulation using Korean and Columbian data, the authors argue that the TFP losses in those countries are, in fact, relatively small (less than 10%). The main mechanism at play is that firms which are more productive are able to quickly accumulate internal funds and grow out of their borrowing constraint. Moll (2012) [3] stresses the importance of persistence of productivity shocks in this process: in the absence of persistence, there is no scope for self-financing, and productivity losses might be higher.

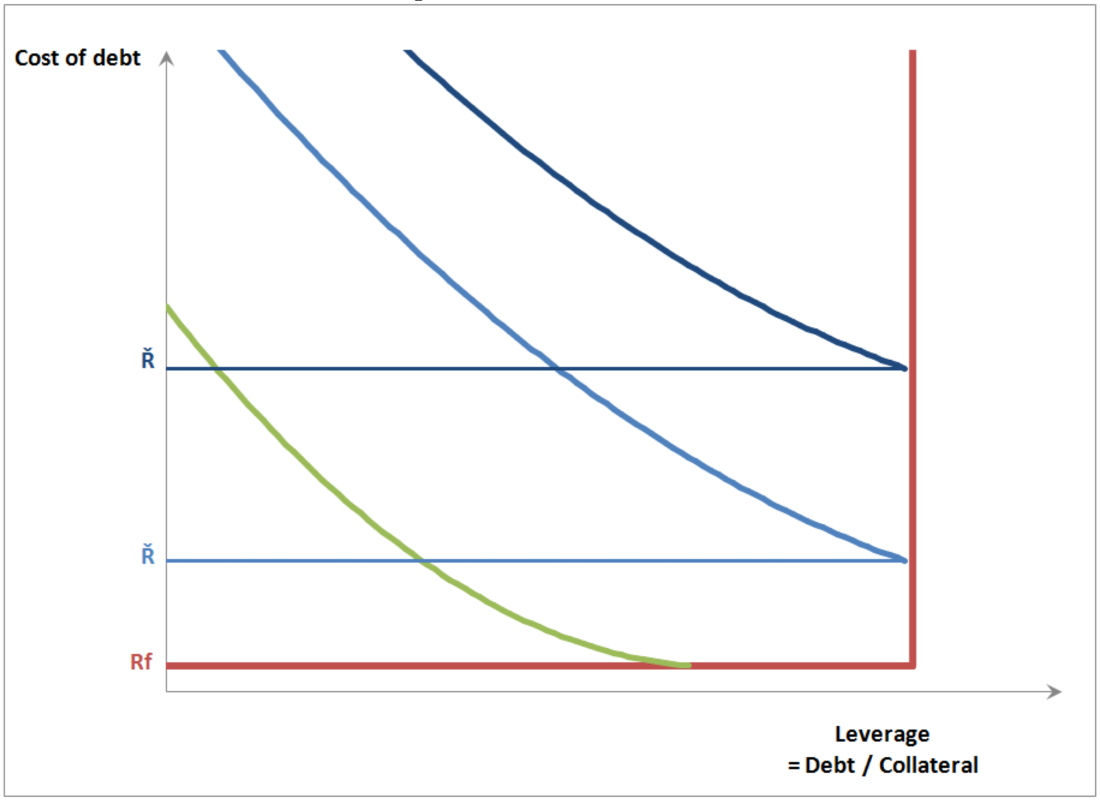
In this literature, binding financial constraints imply that the shadow cost of external funds is higher than the risk-free interest rate, as pictured in Figure 1. Indeed, an entrepreneur willing to borrow more than allowed by the constraint acts as if it were facing a higher cost of credit. However, it should be noted that the actual interest rate paid on debt is always equal to the risk-free interest rate. This simplification comes at a cost, since some productive entrepreneurs may not have access to such cheap credit, and their higher interest payments might hinder the accumulation of retained earnings, thus slowing down the process through which they should grow out of their borrowing constraint. As Midrigan and Xu recognize it themselves, *"finance [...] may still play an important role in distorting allocations if entrepreneurs differ in the interest rates at which they can borrow. If some entrepreneurs have access to cheap credit and others do not, the TFP losses from misallocation* *can be quite large. [...] Whether such differences in terms of financing can indeed account for the large cross-country TFP differences is an open empirical question."*

Parallel work in macroeconomics and finance have studied the interaction of credit spreads and macroeconomic variables such as investment, employment or GDP. Gourio (2011) [4] shows that corporate debt, which is usually safe in normal times, can be subject to price drops in economic depressions, and thus can play an amplifying role in macroeconomic fluctuations. This line of work focuses mostly on medium-run variations in borrowing costs and macroeconomic variables, and does not pay too much attention to the heterogeneity of firms and the efficiency losses resulting from varying costs of debt.

Finally, some papers have focused on entry and exit decisions made by entrepreneurs, and whether their initial wealth is an important determinant for this choice. For example, Hurst and Lusardi (2004) [5] argue that wealth influences the decision to become an entrepreneur, only for the right tail of the wealth distribution, regardless of the capital requirements specific to the industry. They do show that inheritance is, however, a strong predictor for business entry, which can be interpreted as some anecdotal evidence that financial constraints might still be important. Cagetti and De Nardi [6] highlight the importance of savings and bequests in this choice: since firms are constrained in the amount of external funds they can access, entrepreneurs save to grow out of their borrowing constraint, and inheritance is an important part in this mechanism. Their model is successful at explaining the right tail of the wealth distribution, but the implications for smaller businesses are not extensively discussed. Pugsley (2011) [7] further explores the determinants of this occupational choice and finds that tastes for business ownership might matter: some individuals choose to become small business owners, in spite of lower entrepreneurial skills. Not accounting for differences in tastes might lead to an overestimate of the tightness of borrowing constraints. However, to my knowledge, there is no study highlighting the important of heterogeneous borrowing costs in occupational choices. In particular, potential entrants, might face higher costs of borrowing, which might hinder their ability to make their business grow, and explain why some firms remain small.

In all, the consequences of heterogeneous costs of external financing on the allocation of resources has not been studied in a very systematic way. An empirical paper by Gilchrist, Sim and Zakrajsek (2012) [8] does provide an accounting framework to address this issue, and concludes to a small loss from misallocation in the United States (around 2%), but their empirical approach only contains information on bonds quoted on secondary markets, and might be significantly biased in this respect,

Figure 1: Credit constraints



since smaller firms reportedly do not have access to such financial products.

1.2 Policy questions

The theoretical question of selection and resource misallocations from heterogeneous borrowing costs also has important policy implications. In most developed countries, governments subsidize loans to small and medium enterprises (SMEs), either directly or through publicly owned institutions, which either use taxpayers' money or capture a fraction of their savings through various financial instruments.

In Europe, the involvement of European institutions or national government is sizeable. The European Investment Bank (EIB) has used a network of intermediary banks to distribute 7.4 billion dollars of new loans in 2017. Other programs such as the Competitiveness and Innovation Framework Program (CIP) or the European Investment Fund (EIF) also play a role in helping the financing of specific projects led by smaller enterprises, either through direct loans, or through guarantees.

In France alone, BPI France intervenes widely in the credit market, either in the form of direct co-financing alongside banks (7.2 billion euros at end-2017), of loan guarantees (9 billion euros) or direct short-term loans (5.9 billion euros).

Analyzing the benefits of such programs, and in particular the potential efficiency gains of allowing smaller businesses to access cheaper credit, requires that we have a better understanding of how financial constraints and higher borrowing costs impact the entry and exit decisions of firms, as well as the usage of factors by operating enterprises.

This work is a first attempt to tackle those issues. Section 2 presents some motivating data and, specifically introduces a dataset compiled by the Banque de France (French central bank), which I have been granted access to, and which could be used both to identify what drives the heterogeneity in borrowing costs, and to calibrate a model presented in Section 3. This analytic framework endogenously generates firm-specific continuous credit supply curves and heterogeneous borrowing costs, and allows to examine and quantify the aggregate productivity losses generated by firm-specific financial constraints. Section 4 and 5 present a calibration of the model and discusses some preliminary results.

2 Motivating data

In this section, I present some motivating data, that tends to show that the cost of credit can widely differ across firms, and that credit spreads are related to firm size (the larger the firm, the lower the cost of credit) and leverage (the higher the debt as a fraction of total assets, the higher the cost of credit).

2.1 Data

The Banque de France gathers individual loan-level data from a representative sample of French credit institutions. For each loan, the issuing bank reports the loan's precise purpose (investment, cash, leasing), amount, interest rate and maturity, as well as the existence of a private or public subsidy on the loan and various other characteristics. The data is reported on a quarterly basis and covers the 2006-2018 period. About 200,000 new loans to corporations and sole proprietorships (i.e. enterprises owned exclusively by a natural person) are reported every quarter, covering about two thirds of new loan issuance.

The loan-level data is enriched with firm-level characteristics from the Banque de France's FIBEN dataset. FIBEN provides firms' income statements and balance sheets, as well as information on their creation date, age, sector, and whether it belongs to a group or it is a standalone company. The data covers all companies conducting business in France, whose turnover or bank debt exceeds 750,000 or 380,000 euros, respectfully.

FIBEN also includes firm-level credit-ratings, reflecting the Banque de France's assessment of the firm's ability to meet its credit payments at a three-year horizon. We distinguish here between investment-grade firms, which can be used as collateral in Eurosystem refinancing operations, and speculative firms. Speculative firms are characterized by the Banque de France as firms who show some weaknesses in their profitability, solvability, and liquidity ratios and whose ability to meet their payemnts is "rather weak" at best.

2.2 Heterogeneity in the cost of credit across firms

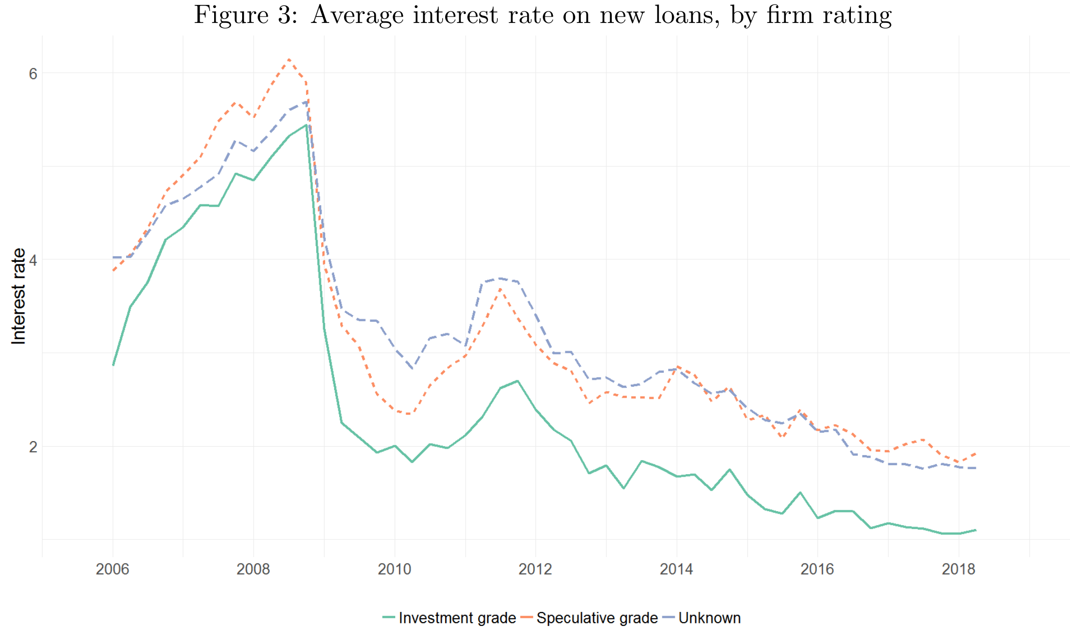
Figure 2 shows the average interest rate charged by banks to firms of various size classes [1]. The measure of interest rate includes the proportion related to the yield to maturity, as well as various charges which mostly consist of all commissions (brokers included) but also compulsory loan insurance premiums against death or unemployment of the borrower. While the series across different firm sizes move together across time, there are sizeable and persistent differences in levels. The larger the firm, the lower the interest rate charged by banks on average. On average, microenterprises pay a 150bp premium on new loans compared to large enterprises.

In the same way, figure 3 shows the average interest rate charged by banks to firms of various ratings classes. Speculative grade firms and firms for which the credit rating is not available pay a 70bp premium over investment grade firms, on average across the period. This spread has substantially increased in the aftermath of the 2008 crisis, and has slightly decreased since then.

[1] According to the French statistical office (Insee), the French law specifies four categories of business (Article 51 of the law on the modernisation of the economy) for the purposes of statistical and economic analysis. To determine the category to which a business belongs, the following items of information are used, pertaining to the most recent complete trading year ended and calculated on an annual basis: headcount, turnover and balance sheet total. The category of Microenterprises consists of enterprises which employ less than 10 persons and either have an annual turnover or a total balance sheet not exceeding 2 million euros. The category of Small enterprises consists of enterprises which employ less than 250 persons and either have an annual turnover not exceeding 50 million euros or a balance sheet total not exceeding 43 million euros. The category of Medium-sized enterprises consists of enterprises which do not belong to the category of small and medium-sized enterprises and which employ less than 5,000 persons and either have an annual turnover not exceeding 1,500 million euros or a balance sheet total not exceeding 2,000 million euros. The Large enterprises category consists of enterprises that are not classified in the previous categories.

Figure 2: Average interest rate on new loans, by firm size

Figure 3: Average interest rate on new loans, by firm rating



Our rich database allows to go beyond simple averages and to distinguish the impact of various loan and firm-level characteristics on the cost of credit. In particular, table 1 shows that there might be a systematic relationship between the interest payments and the size of the firm, controlling for other loan and firm characteristics (columns 1, 3, and 5), and allowing for these controls to vary over time (columns 2, 4, and 6). The results of the regression tend to show that, everything else being equal (including the firm's acknowledged credit-worthiness), medium-sized, small and microenterprises pay a 60bp, 110bp and 190bp premium on bank loans compared to larger enterprises, respectively. The results are highly statistically significant and robust across all years in the sample, as shown in table 8 of Appendix A.

The effect is sizeable when the sample is restricted to investment loans, with higher spreads for medium-sized enterprises and lower spreads for microenterprises. The effect is slightly lower for short-term cash loans, but remains statistically significant.

Table 2 shows that, even when controlling for firm size, there is a positive relationship between the interest rate charged by banks, and the initial stock of debt held by a firm: a 1% increase in the debt/assets ratio is associated with a 8bp increase in the interest rate. The result holds for investment loans, but not for cash loans (not reported). This suggests that firms which go deeper into debt usually do it at a higher cost. It may correspond to the fact that, with uncertain cash flows, the return on debt for lenders is risky and the price of corporate debt diminishes with the amount of debt issued. It should be noted that the slope coefficient is higher for microenterprises than for larger companies (it is virtually negative for large enterprises, but the coefficient is not quite as significant).

Table 1: Interest rate on new loans and firm size, regression results by loan type

	Dependent variable:					
	Interest rate on new loans					
	All loans		Investment loans		Cash loans	
	(1)	(2)	(3)	(4)	(5)	(6)
Medium-sized enterprises	0.653***	0.648***	1.077***	1.124***	0.192***	0.132***
	(0.004)	(0.005)	(0.014)	(0.013)	(0.012)	(0.012)
Small enterprises	1.103***	1.089***	1.069***	1.117***	0.436***	0.429***
	(0.004)	(0.004)	(0.014)	(0.013)	(0.012)	(0.012)
Microenterprises	1.920***	1.876***	1.429***	1.453***	1.334***	1.276***
	(0.005)	(0.005)	(0.014)	(0.013)	(0.014)	(0.014)
Quarter FE	Yes	Yes	Yes	Yes	Yes	Yes
Firm sector FE	Yes	Yes	Yes	Yes	Yes	Yes
Firm rating FE	Yes	Yes	Yes	Yes	Yes	Yes
Firm size FE	Yes	Yes	Yes	Yes	Yes	Yes
Loan amount FE	Yes	Yes	Yes	Yes	Yes	Yes
Loan maturity FE	Yes	Yes	Yes	Yes	Yes	Yes
Loan type FE	Yes	Yes	No	No	No	No
Loan type FE	Yes	Yes	Yes	Yes	Yes	Yes
Year x Firm rating FE	No	Yes	No	Yes	No	Yes
Year x Firm size FE	No	Yes	No	Yes	No	Yes
Year x Loan amount FE	No	Yes	No	Yes	No	Yes
Year x Loan maturity FE	No	Yes	No	Yes	No	Yes
Year x Loan type FE	No	Yes	No	No	No	No
Observations	2,323,037	2,323,037	506,131	506,131	283,590	283,590
Adjusted R^2	0.620	0.646	0.664	0.724	0.632	0.644

*p<0.1; **p<0.05; ***p<0.01

Table 2: Interest rate on new loans and debt/assets ratio, regression results by firm size

	Dependent variable:									
	Interest rate on new loans Investment loans									
	All enterprises		Microenterprises		Small enterprises		Medium-sized enterprises		Large enterprises	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Debt/Assets ratio	0.086***	0.036***	0.267***	0.115***	0.225***	0.141***	0.159***	0.022	−0.487***	−0.195**
	(0.009)	(0.008)	(0.030)	(0.026)	(0.012)	(0.011)	(0.018)	(0.016)	(0.075)	(0.082)
Quarter FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm sector FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm rating FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Loan amount FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Loan maturity FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year x Firm rating FE	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes
Year x Loan amount FE	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes
Year x Loan maturity FE	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes
Observations	434,694	434,694	42,463	42,463	270,286	270,286	117,958	117,958	3,987	3,987
Adjusted R^2	0.662	0.738	0.625	0.720	0.673	0.745	0.659	0.727	0.735	0.761

*p<0.1; **p<0.05; ***p<0.01

Our dataset contains information on the existence of a subsidy for each individual loan. The subsidy could come from a public entity (European, national or local government) or a private agent (most notably publicly-owned banks). It can take the form of a direct subsidy associated with the loan operation, or a guarantee, which lowers the observed or implicit cost of credit. Table 3 highlights that the existence of a subsidy is usually associated with a lower cost of credit (of about 17bp for investment loans, and 82bp for cash loans). The effect is a lot larger for microenterprises (61 and 149bp on investment and cash loans, respectively) and cancels out as firms are larger. For large firms, subsidies on investment loans are typically associated with larger costs of credit, possibly reflecting the fact that subsidies might be associated with riskier credits and a poorer selection of investment projects.

Table 3: Interest rate on new loans and loan subsidies, regression results by firm size and loan type

	Dependent variable:			
	Interest rate on new loans			
	Investment loans		Cash loans	
	(1)	(2)	(3)	(4)
Subsidy	-0.171^{***}		-0.818^{***}	
	(0.004)		(0.012)	
Subsidy x Microenterprise		-0.608^{***}		-1.489^{***}
		(0.008)		(0.016)
Subsidy x (Medium-sized enterprise - Microenterprise)		0.416^{***}		1.230^{***}
		(0.008)		(0.022)
Subsidy x (Small-sized enterprise - Microenterprise)		0.647^{***}		1.453^{***}
		(0.009)		(0.048)
Subsidy x (Large enterprise - Microenterprise)		0.910^{***}		1.315^{***}
		(0.076)		(0.156)
Quarter FE	Yes	Yes	Yes	Yes
Firm sector FE	Yes	Yes	Yes	Yes
Firm rating FE	Yes	Yes	Yes	Yes
Loan amount FE	Yes	Yes	Yes	Yes
Loan maturity FE	Yes	Yes	Yes	Yes
Year x Firm rating FE	Yes	Yes	Yes	Yes
Year x Loan amount FE	Yes	Yes	Yes	Yes
Year x Loan maturity FE	Yes	Yes	Yes	Yes
Observations	506,131	506,131	283,590	283,590
Adjusted R^2	0.725	0.728	0.650	0.655

*p<0.1; **p<0.05; ***p<0.01

Table 4: Regression of firm growth on size and interest payments

Dependant variable:	Sales growth		Employment growth		Assets growth		Physical capital growth	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Log(Sales)	-0.767***	-0.807***						
	(0.083)	(0.083)						
Log(Employment)			-0.156***	-0.159***				
			(0.015)	(0.015)				
Log(Total assets)					-0.059***	-0.062***		
					(0.004)	(0.004)		
Log(Total physical capital)							-0.244***	-0.259***
							(0.016)	(0.016)
Interest payment / Debt		-0.353***		-0.027**		-0.021***		-0.098***
		(0.055)		(0.010)		(0.002)		(0.011)
Constant	4.177***	7.447***	0.166***	0.399***	0.263***	0.465***	1.111***	2.028***
	(0.437)	(0.675)	(0.028)	(0.091)	(0.019)	(0.030)	(0.077)	(0.132)
CONTROL	YES	YES	YES	YES	YES	YES	YES	YES
R-squared	0.215	0.215	0.087	0.087	0.025	0.026	0.078	0.079

* p<0.05, ** p<0.01, *** p<0.001

2.3 Cost of credit and firm growth

Finally, table 4 highlights that firm growth is negatively correlated with interest payments. That is to say that, everything else being equal, firms who pay higher interest rates tend to grow more slowly in the future.

For now, I am not suggesting any causal interpretation: growth might be specifically hindered for the firms facing high borrowing costs, or smaller firms may be charged higher costs because their activity and growth prospects are usually more uncertain then for larger firms. My model will embed both interpretations.

3 Model with heterogeneous entrepreneurs and endogenous cost of credit

The model developed here borrows from Midrigan and Xu (2010) [2] and subsequent papers dealing with misallocations induced by financial constraints. It extends the usual setup by incorporating the possibility that firms face heterogeneous costs of credit, which are endogenously determined in the model and depend on both firms' characteristics and choices. The model abstracts from issues related to asymmetry of information: all agents, lenders and borrowers, discover the same information at the same time. Future extensions might include a better way of dealing with information asymmetries that might be an important factor when considering bond prices and credit spreads.

The mechanisms for pricing loans is inspired by Gourio (2011) [4]: borrowers can collateralize the depreciated capital bought with debt, as well as their profits. But since profits are uncertain, due to the stochastic nature of productivity, so is the expected payoff, which drives the price of the bond to higher levels than the risk free rate. Finally, my setup incorporates discrete occupational choices made by agents, between entrepreneurship and the labor market, much like in Hopenhayn (1992) [10] and Pugsley (2011) [7].

The next subsections describe the setup of the model, and illustrate the basic mechanisms at play.

3.1 General settings

The model features a unit measure of households, indexed by $i \in [0, 1]$. Each household is infinitely lived and has preferences over a single, final good. Her intertemporal utility function is characterized by a constant relative risk aversion:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{C_{it}^{1-\gamma}}{1-\gamma} \right]$$

where C_{it} is the individual's consumption level at date t, $0 < \beta < 1$ the discount factor, and $0 < \gamma$ represents both the coefficient of relative risk aversion, and the inverse of the intertemporal elasticity of substitution. Note that the limit case where $\gamma = 1$ corresponds to logarithmic preferences.

Each household is endowed with an indivisible unit of labor, supplied inelastically. At the very beginning of each period, the household gets to choose whether it wants to allocate its unit of labor to being a worker (W), or to becoming an entrepreneur and running a business (E). While the choice between working and running a business is binary, the unit of labor of a worker can be divided among various businesses at no cost.

Individuals differ in their entrepreneurial skills, which measure their ability to run a business and the productivity of the firms they create. These skills vary over time but the process governing entrepreneurial abilities for each individual entrepreneur is persistent.

If an individual chooses to become a worker, she earns wage W_t, gets to consume and save, and enters the next period with some assets. The individual is then free to choose whether it remains a worker in the subsequent period, or if she becomes an entrepreneur.

The entrepreneur's problem is slightly more complex, and I devote the next subsection to describing the main decisions it faces.

3.2 Entrepreneur's problem

The following paragraphs describe the assumptions regarding technology and timing, and derive the equations for the credit supply curve faced by entrepreneurs. The dynamic problem will be studied in the next subsection

3.2.1 Productivity and production

Each individual who runs a firm has access to the same technology, described by a Cobb-Douglas production function. This technology uses capital and labor to produce a homogeneous good. More precisely, the production function is given by:

$$Y_{it} = z_{it} \left(K_{it}^{\alpha} L_{it}^{1-\alpha} \right)^{\eta}$$

where Y_{it} denotes the output of firm i at date t, K_{it} its capital stock, L_{it} labor demand, $0 < \alpha < 1$ the share of capital in the production function, $0 < \nu < 1$ the degree of decreasing returns to scale (or span of control), and z_{it} is the entrepreneur's specific productivity parameter.

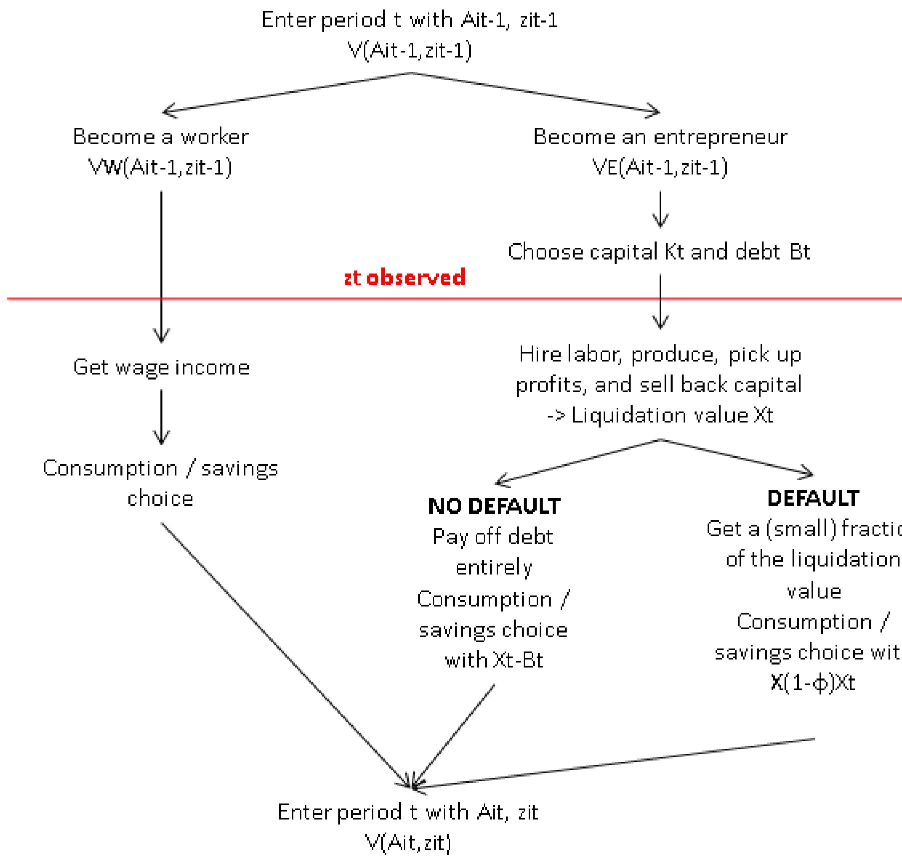
Productivity is assumed to follow a Markov process: current productivity only depends on observed productivity in the last period, and an i.i.d. contemporaneous shock.

3.2.2 Timing and notations

Figure 4 describes the timing assumption. Each individual who chooses to become an entrepreneur enters period t with some assets, A_{it-1}. At the very beginning of the period, before the lender observes the realization of the entrepreneur's specific productivity shock, the firm has to invest in new capital K_{it}, which might be financed either by its retained earning, or by contracting new debt. The price of the bond issued by the firm is denoted q_{it}, while the amount of debt to be payed back by the end of the period is denoted B_{it}.

After the investment and debt decisions are made, the productivity shock is realized, and the firm hires labor L_{it} at a wage W_t, to generate profits. It also formally hires the entrepreneur and gives him a fraction ω of the wage rate. This can be thought of as a minimal level of income that should

Figure 4: Timing assumption



be transferred to the entrepreneur, and that cannot be seized by the lender. In other words, the entrepreneur's total income is made out of labor income, which is certain, predetermined and lower that of a worker, and dividends, which are risky and depend on the realization of the entrepreneur's specific productivity shock.

Before the end of the period, and after profits are generated, the firm has to repay its debt. If the firm's assets, composed of its profits and undepreciated capital, are enough to cover the face value of the debt, the firm repays its debt entirely, and then decides how much dividends the entrepreneur can obtain, and how much earnings to retain in its balance sheet. If the firm's assets are not sufficient to repay its debt and the entrepreneur's wage income, then the firm defaults and is liquidated. A fraction χ of its assets is split between all bondholders, and the entrepreneur receives no dividend.

Before studying the intertemporal aspects of the agent's decision, I focus on the intratemporal choices of the entrepreneur, regarding debt and capital. As usual, I solve the problem "backwards".

3.2.3 Profits and assets

Once the shock is realized, the firm gets to maximize profits by choosing how much labor to employ. Its maximization problem writes:

$$\begin{aligned}\pi_{it}(K_{it}, z_{it}, W_t) &= \max_{L_{it}} \quad \{(1-\tau_\pi)(Y_{it} - W_t L_{it} - \omega W_t)\} \\ &= \max_{L_{it}} \left\{(1-\tau_\pi)(z_{it}\left(K_{it}^{\alpha} L_{it}^{1-\alpha}\right)^{\eta} - W_t L_{it} - \omega W_t)\right\}\end{aligned}$$

Taking first order conditions, it is straightforward to show that labor and profits are given by the following expressions:

$$\begin{aligned}L_{it}(K_{it}, z_{it}, W_t) &= \left(\frac{(1-\alpha)\eta}{W_t} z_{it} K_{it}^{\alpha\eta}\right)^{\frac{1}{1-(1-\alpha)\eta}} \quad (1)\\ \pi_{it}(K_{it}, z_{it}, W_t) &= (1-\tau_\pi)\left[(1-(1-\alpha)\eta)\left(\frac{(1-\alpha)\eta}{W_t}\right)^{\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}} (z_{it} K_{it}^{\alpha\eta})^{\frac{1}{1-(1-\alpha)\eta}} - \omega W_t\right]\end{aligned}$$

Obviously, both labor and profits are increasing in the contemporaneous productivity parameter, and in the level of the capital stock, decided at the beginning of the period.

The firm's liquidation value at the end of the period consists of the firm's profits, and its undepreciated capital stock:

$$\mathcal{L}_{it}(z_{it}, K_{it}, W_t) = \pi_{it}(K_{it}, z_{it}, W_t) + (1-\delta)K_{it}$$

3.2.4 Default and cost of credit

Once production and profits are realized, the entrepreneur chooses whether she wants to pay back her debt, or whether she wants to default. In the case of no default, the entrepreneur pays back her debt, and keeps the net liquidation value of the firm to herself. In the case of default, the firm loses a fraction $(1-\chi)$ of its value, corresponding to bankruptcy costs, and the value after bankruptcy is split between the lender (who recovers a fraction phi) and the entrepreneur (fraction $(1-\phi)$). The value and payoffs under default and no default are given in table 5.

Equivalently, the firm will default if the entrepreneur's productivity draw is below some threshold

Table 5: Default and no default

	No default	Default
Total value	$\mathcal{L}_{it}(z_{it}, K_{it}, W_t)$	$\chi\mathcal{L}_{it}(z_{it}, K_{it}, W_t)$
Entrepreneurs	$\mathcal{L}_{it}(z_{it}, K_{it}, W_t) - B_{it}$	$(1-\phi)\chi\mathcal{L}_{it}(z_{it}, K_{it}, W_t)$
Lenders	B_{it}	$\phi\chi\mathcal{L}_{it}(z_{it}, K_{it}, W_t)$

value, which I denote $\underline{z}_{it}$:

$$\underline{z}_{it} = \frac{(\omega W_t + B_{it} - (1-\delta)K_{it})^{1-(1-\alpha)\eta}}{K_{it}^{\alpha\eta}} \frac{W_t^{(1-\alpha)\eta}}{[(1-\alpha)\eta]^{(1-\alpha)\eta}[1-(1-\alpha)\eta]^{1-(1-\alpha)\eta}} \tag{2}$$

Note that the threshold is increasing in the amount of debt contracted by the firm, and decreasing in the capital stock of the firm at the end of the period.

Now, the expected payoff for a bondholder is given by:

$$\chi \int_0^{\underline{z}_{it}} \frac{\mathcal{L}_{it}(z_{it}, K_{it}, W_t)}{B_{it}} d\mathbb{F}(z) + \int_{\underline{z}_{it}}^{\infty} 1 d\mathbb{F}(z)$$

The term corresponding to the first integral accounts for all the instances where the firm defaults, and the bondholders share a fraction χ of the liquidation value of the firm. The other terms corresponds to all other instances, where the firm does not default and is able to pay each bondholder one unit of good, as promised. The parameter $1-\chi$ represents the cost of bankruptcy, which the corporate finance literature has identified as being the fraction of the firm's value which is lost in case it faces bankruptcy. This loss encompasses a variety of different factors, like legal or auditor's fees.

If the world is populated by risk neutral investors, the price of the bond will be equal to the discounted expected payoff, which can be rewritten, after some omitted algebra steps, with the following formula:

$$\begin{aligned} q_{it} &= \beta\chi\mathbb{E}\left[\left(\frac{z}{\underline{z}_{it}}\right)^{\frac{1}{1-(1-\alpha)\eta}} | z < \underline{z}_{it}\right] \\ &+ \beta\chi\left(\mathbb{F}[\underline{z}_{it}] - \mathbb{E}\left[\left(\frac{z}{\underline{z}_{it}}\right)^{\frac{1}{1-(1-\alpha)\eta}} | z < \underline{z}_{it}\right]\right)\frac{(1-\delta)K_{it} - \omega W_t}{B_{it}} + \beta\left(1 - \mathbb{F}[\underline{z}_{it}]\right) \end{aligned} \tag{3}$$

An illustration for the bond price is given in appendix B.

3.3 Dynamic problem and Bellman equation

Now, I can study the dynamic problem of an individual, who enters period t with some wealth A_{it-1} and entrepreneurial skills z_{it-1}. I denote $\mathcal{V}$ the value function of such an individual, $\mathcal{V}^W$ the value function of a worker, and $\mathcal{V}^E$ the value function of an entrepreneur. Obviously, the choice of becoming a worker or an entrepreneur will depend on those two value, so that:

$$\mathcal{V}(A_{it-1}, z_{it-1}) = \max_{J \in W,E} \left(\mathcal{V}^W(A_{it-1}, z_{it-1}), \mathcal{V}^E(A_{it-1}, z_{it-1}) \right) \tag{4}$$

Now, a worker who chooses to become a worker gets a wage W_t and his savings, augmented with interest, at the beginning of the period. She decides how much to consume and save. In period $t+1$, it will get to decide again on whether it wants to work on run a business. Therefore, the value function for a worker is given by:

$$\mathcal{V}^W(A_{it-1}, z_{it-1}) = \int_{z=0}^{\infty} \left\{ \begin{array}{ll} \max\limits_{C_{it}(z), A_{it}(z)} & \frac{C_{it}(z)^{1-\gamma}}{1-\gamma} + \beta \mathcal{V}(A_{it}(z), z) \\ \text{s.t.} & A_{it}(z) + C_{it}(z) = (1+r_t)A_{it-1} + W_t \end{array} \right\} d\mathbb{F}(z|z_{it-1}) \tag{5}$$

The problem for an agent who decided to become an entrepreneur is somehow more complicated. At the beginning of the period, it has to decide how much capital to buy, subject to the fact that capital and his wage income have to be financed with retained earnings and debt. Then the productivity shock is realized and two cases arise. If it is below the cutoff value $\underline{z}_{it}$, the entrepreneur will default on his debt and be excluded from financial markets for one period, which means it will carry no assets in the next period, will not have access to credit, and will be forced to become a worker. In the case where the productivity draw is above the threshold, the entrepreneur may choose to retain a dividend, and will carry the remaining of its assets, after debt repayments, to the next period. Its value function writes:

$$
\begin{aligned}
&\mathcal{V}^E(A_{it-1}, z_{it-1}) \\
&= \max_{K_{it},B_{it}} \int_{z=0}^{\underline{z}_{it}} \left\{ \frac{(\omega W_t)^{1-\gamma}}{1-\gamma} + \mathcal{V}^W(0,z) \right\} d\mathbb{F}(z|z_{it-1}) \\
&\quad + \int_{z=\underline{z}_{it}}^{\infty} \left\{ \begin{array}{l} \max\limits_{C_{it}(z),A_{it}(z)} \frac{C_{it}(z)^{1-\gamma}}{1-\gamma} + \beta\mathcal{V}(A_{it}(z), z) \\ \text{s.t. } A_{it}(z) + C_{it}(z) = \pi(z_{it}, K_{it}, W_t) + \omega W_t + (1-\delta)K_{it} - B_{it} \end{array} \right\} d\mathbb{F}(z|z_{it-1}) \\
&\text{s.t. } K_{it} + \omega W_t \leq q_{it} B_{it} + A_{t-1}
\end{aligned} \tag{6}
$$

3.4 Stationary distribution

My definition of an equilibrium involves a stationary distribution ψ on assets and productivity, that I can derive using the policy functions above, as well as the exogenous evolution of productivity. This invariant distribution, is defined as a fixed point by the following equation:

$$
\psi(A', z') = \int_{A,z} \mathbb{I}_{\mathcal{A}(A,z,z')=A'} \psi(dA, dz) \mathbb{F}(z'|dz) \tag{7}
$$

3.5 Recursive equilibrium

To close the model, I jointly determine the wage rate W_t and the interest rate on savings r_t, such that the labor and asset markets clear, given the stationary distribution and policy functions found above.

Labor market clearing is achieved by assuming that the supply of labor by workers (which includes the former entrepreneurs who defaulted, have no savings, and are excluded from the borrowing market) is equal to the demand of labor by current entrepreneurs:

$$\int\limits_{A,z,z'} \mathbb{I}_{\mathcal{J}(A,z)=E}\mathcal{L}(dA,dz,dz')\psi(dA,dz)\mathbb{F}(dz'|dz) = \int\limits_{A,z,z'} 1.\mathbb{I}_{\mathcal{J}(A,z)=W}\psi(dA,dz)\mathbb{F}(dz'|dz) \tag{8}$$

Clearing the market for assets requires that the savings of the workers should be equal to the total amount of funds borrowed by the entrepreneurs in the economy. This is saying that some financial institution collects the savings from the workers, and redistributes it to the entrepreneurs. The market for banking intermediation is competitive, so this financial institution makes no profit in equilibrium. In all, we have:

$$\int\limits_{A,z} A\psi(dA,dz) = \int\limits_{A,z} \mathbb{I}_{\mathcal{J}(A,z)=E}\,q(A,z)\mathcal{B}(A,z)\psi(dA,dz) \tag{9}$$

I can now define a recursive stationary equilibrium.

Definition. *Given a set of parameters and a transition function for the distribution of skills, a recursive stationary equilibrium consists of:*

- *A set of value functions for the individual $\mathcal{V}(A,z)$, the worker $\mathcal{V}^W(A,z)$, and the entrepreneur $\mathcal{V}^E(A,z)$, and a set of policy functions for the occupational choice, $\mathcal{J}(A,z)$, capital $\mathcal{K}(A,z)$, debt $\mathcal{B}(A,z)$, bond prices $q(A,z)$ and future assets $\mathcal{A}(A,z,z')$ such that the Bellman equations are satisfied (Equations 4, 5 and 6), as well as the bond pricing equations (2 and 3)*
- *A policy function $\mathcal{L}(A,z,z')$ that maximizes profits (Equation 1).*
- *An invariant distribution over all individuals ψ (Equation 7).*
- *Constant prices for labor W_t and r_t such that markets clear (Equation 8 and 9).*

4 Simulation

To examine the role of financial constraints and heterogeneous costs of credit, I calibrate and simulate the model described above. This section describes the choice of parameter values, the algorithm used to simulate the model, and discusses a few preliminary results.

4.1 Parameters

Table 6 contains the values of the calibrated parameters of the model. Most values are close to the ones used by Midrigan and Xu. The parameters governing the production function (share of capital, span of control, depreciation rate) are standard in the literature. The preferences parameters that I consider are slightly off the baseline Midrigan and Xu specification, but closer to the ones used in the macroeconomic literature. In particular, I choose a higher discount factor (0.95 compared to 0.92) and a higher coefficient of relative risk aversion (2, compared to 1). The persistence and standard deviation of the productivity process are the same, and the unconditional mean of productivity is normalized to one.

More importantly, some parameters do not appear in the "traditional" literature on finance and misallocations. In particular, I choose the recovery rate on the liquidation value for lenders to be 0.7, as in Gourio [4]. This means that 30% of the value of a firm is lost when it defaults, which roughly falls in the range of values that the corporate finance literature would deem acceptable. Finally, I set the parameter ω, which measures the fraction of the wage rate paid to the entrepreneur, to be 0.7. I do not have any priors on what this value could be. For now, I set it to obtain a proportion of individuals becoming entrepreneurs of around 5%.

To solve the problem numerically, I make the space for state and decision variables discrete. While building up grids is fairly straightforward when it comes to endogenous states and controls, making the space of productivity shocks discrete is slightly less obvious. I use a procedure described by Tauchen and implemented by Pugsley (2011) [7] to approximate the AR(1) by a discrete Markov chain.

Note that, while building grids for the state and control spaces is costly in all numerical procedures, it is especially true in my problem, where small variations in either debt or capital can have a dramatic impact on the price of the bond, and hence on the allocation that will be chosen optimally. Evidently, there is a balance between the size of the grids and the computational costs, which I have not quite resolved yet. Other numerical procedures (endogenous grids, etc) might be explored in the future.

Table 6: Simulation parameters

Parameter	Value
α Share of capital in the production function	0.33
η Returns to scale	0.85
δ Depreciation rate	0.06
β Discount factor	0.95
γ Coefficient of relative risk aversion	2.00
ρ Persistence of the log-productivity process	0.91 in France, 0.90 in the US
σ Standard deviation of the log-productivity process	0.68 in France, 0.65 in the US
χ Recovery rate on liquidation value	0.62 in France, 0.70 in the US
ω Fraction of the wage income earned by entrepreneurs	0.70

4.2 Algorithm

For now, my algorithm relies on value function iteration. The first step is to exogenously set a wage rate and an interest rate on savings, and guess a value function for the worker and the entrepreneur.

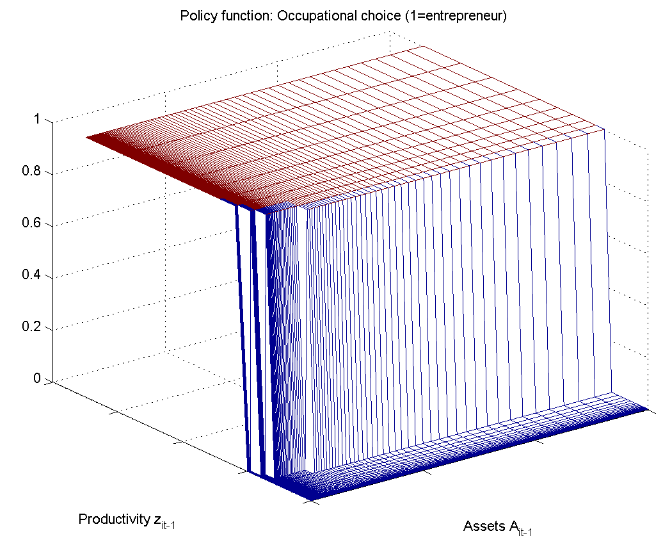
Updating the value function for the worker is relatively easy: the worker only gets to choose between consumption and savings once its productivity shock is observed. Hence, I only need to keep track of each state-dependant allocation, and sum across all states.

Updating the value function for the entrepreneur is a little trickier, since this type of agents needs to choose a level of debt and capital before the productivity shock is realized. Hence, for each initial state, I consider all the feasible pairs of capital and debt. For each of those pairs, I compute the period value function and continuation values, by allowing the agent to maximize on consumption and savings when the realization of the productivity shock is such that it does not default, and by forcing him to consume a fraction of the wage rate if he does default. Again, I sum across all realizations of the shock, and eventually choose the pair of capital and debt that provides the maximum value.

I iterate on the value function until it converges to a fixed point.

With all the value and policy functions, I can compute the stationary distribution as a fixed point as well. From there, I can derive the aggregate demand and supply of labor and assets. The equilibrium is found when, after a final iteration procedure on the wage rate and the interest rate on savings, demand and supply are equalized on both the labor and assets markets.

Figure 5: Policy function - Occupational choice



4.3 Results

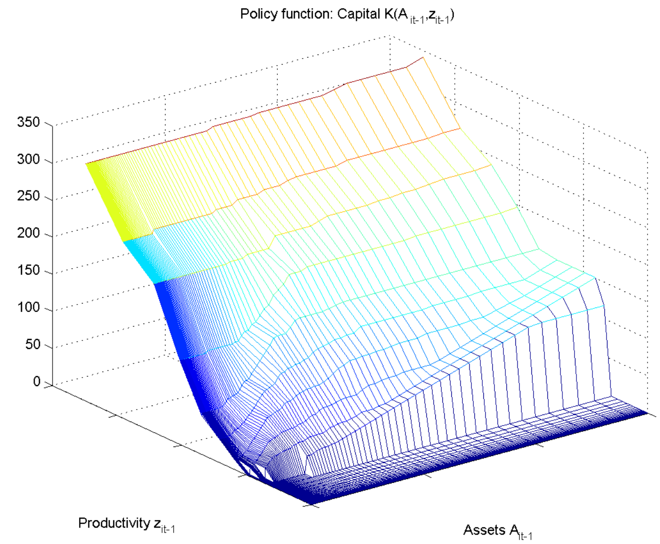
4.3.1 Firm selection

Figure 5 describes the occupational choice made by each individual, as a function of its initial assets and its initial productivity. It is clear that only individuals with a sufficiently high productivity expect to draw a high productivity shock in the current period, and choose to become entrepreneurs. However, the decision is also influenced by current assets. While some skilled entrepreneurs opt out of running a business because they do not have enough savings, or good options to finance their investment through the credit market, some less-skilled entrepreneurs opt in, mostly because financing their capital needs through savings and equity is feasible and cheap. Note that this selection already induces misallocations, even in this world characterized by imperfect information on current productivity, where the constrained efficient allocation would imply that only productive entrepreneurs would be in the market, regardless of the savings they accumulated in the past.

4.3.2 Capital, debt, and cost of credit

The losses from misallocations are even more evident when looking are the choices for capital, displayed in Figure 6. Indeed, capital is increasing in both initial productivity, and initial assets. This comes from the fact that highly skilled entrepreneurs do not have much savings and have to

Figure 6: Policy function - Capital



finance the bulk of their capital expenses through debt. Since only a fraction of capital can be collateralized at no cost, they have to accept higher borrowing costs to finance the remaining stock (7). In equilibrium, the optimal level of capital that they choose is significantly lower than that of similarly productive entrepreneurs which a higher initial stock of assets.

Note that the optimal ratio of the face value of debt to capital increases dramatically for those entrepreneurs who have a relatively low productivity, and close to no savings. Their borrowing costs are significantly higher, since their productivity draws will be, on average, lower than those of initially more productive entrepreneurs.

Figure 8 plots the borrowing cost against the initial level of assets (left panel) and the initial productivity level (right panel). The picture shows that the interest rate is decreasing in both the initial savings of the firm, and the initial productivity draw, which should be related to the initial size of the firm. This prediction of the model is - at least qualitatively - consistent with some observations in the data.

Figure 9 plots the borrowing cost against the optimally chosen ratio of debt to capital. Here, we see that firms which go deeper into debt tend to face higher interest rates, which again seems to be consistent with some empirical observations.

Figure 7: Policy function - Interest rate

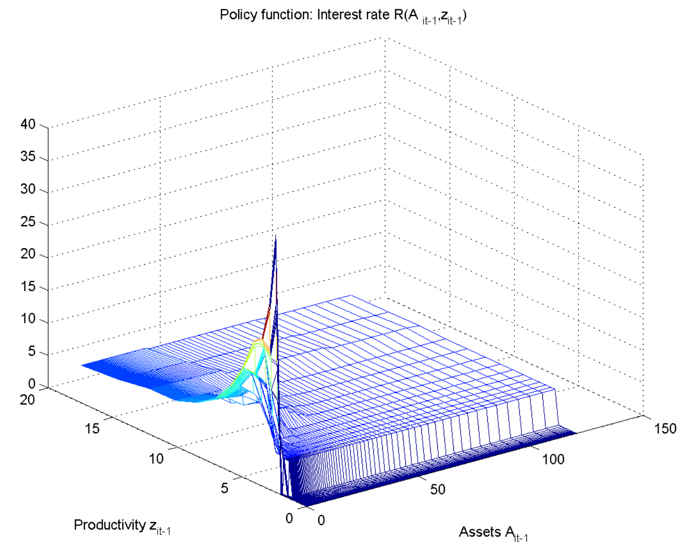


Figure 8: Interest rate

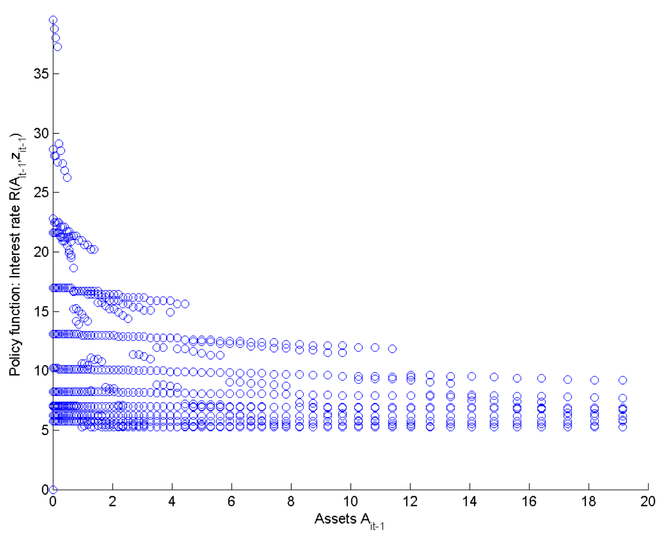
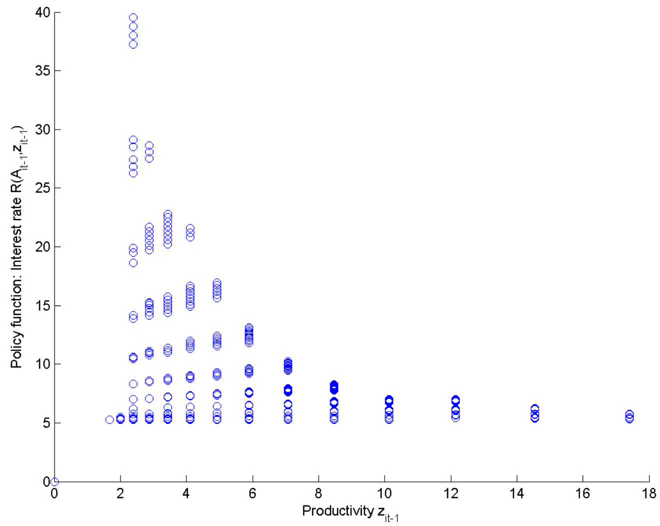


Figure 9: Interest rate



5 Productivity losses from financial frictions and policy experiments

5.1 Productivity losses from financial frictions

I use the model to estimate the effects of financial frictions on aggregate productivity, in France, using two different benchmark economies. The first benchmark is the on exposed in the paper by Midrigan and Xu, where capital would be allocated efficiently by a social planner who observes current productivity shocks. The second benchmark economy is the model economy calibrated to match data on the US.

As shown in Table 7, financial frictions cause TFP losses of about 10% in France compared to a situation where a central planner efficiently allocates capital. Even compared to the model economy calibrated for the US, the TFP losses are still sizable (2%). These estimates are potentially higher than those found in the Midrigan and Xu framework. This is to be related to the structure of our economy, which features both selection mechanisms to entrepreneurship, and slower growth dynamics for smaller firms, due to higher interest payments, both of which are not accounted for by previous papers.

Table 7: Results

TFP losses from financial frictions	
Compared to the frictionless economy	9.6%
Compared to the US	2.1%
Impact of policy experiments on TFP	
Credit subsidy (1% of GDP)	1.1%
Government guarantee on loans	1.5%

5.2 Policy experiments

Finally, I conduct a set of policy experiments designed to facilitate the access of firms to credit, but I assume that the government does not have better information than the private sector regarding individual entrepreneurs. While it cannot condition its actions to agents' current productivity, it can still target its subsidies or guarantees towards a subset of firms, based on their size or other observables.

In order to better match the actual policy experiments that we observe throughout the worlds, I first consider a direct subsidy to credit, designed to lower the spread between the actual interest rate charges, and the risk-free rate by 25% to the 25% smallest firms (based on employment). This subsidy is financed by a simple tax on wage income received by workers. The size and destination of the subsidy are chosen so that the aggregate effort represents 1% of GDP.

The second experiment that I consider is a government guarantee, that diminishes the value loss of firms under distress, so that the riskiness of the loans is diminished. This too is financed by a tax on workers and is calibrated so that the total effort represents 1% of GDP.

The results, shown in table 7 bring two comments. First, the aggregate productivity gain is sizable and above 1%, showing that such policies, designed to alleviate the riskiness of entrepreneurs in the eyes of the lenders and diminish the friction, are efficient. This effect stems from a reduction in the selection bias, where individuals with no internal funds can access cheaper credit and operate as entrepreneurs, and from a rebalancing in factor allocation, to the benefit of productive agents. However, the effect of both policies remains small (it doesn't exceed 1.5%), which comes from the fact that these policies only target entrepreneurs based on the size of the firm thy operate and not based on their current productivity.

6 Conclusion and speculations

My model includes occupational choices and endogenously determined costs of credit, which matches some facts observed in the data. The main prediction of the model is that initial assets matter when it comes to firm selection, and that entrepreneurs which have not accumulated enough savings in the past face higher borrowing costs, which might significantly hinder the wealth accumulation process and force them to either get out of the market, or remain small.

I conduct numerical exercises and show that financial frictions can account for TFP losses of about 10% in France. My model also allows to estimate the impact of different policies designed to facilitate firms' access to credit. I show that credit subsidies and government guarantees on loans only have a moderate impact on TFP, since public authorities do not have better technologies to observe actual productivity and cannot discriminate productive entrepreneurs better than the private sector.

The model, while novel along some dimensions, leaves out some aspects of the relationship between lenders and borrowers. In particular, banks put a lot of resources into screening technologies, that allow them to make better predictions on individuals' productivity, than the ones obtained by just observing the aggregate distribution of productivity shocks. These technologies usually come with a cost and the choice whether to use them or not has been studied in the economic literature. Embedding the other dimension into the model would come with a computational cost, but could allow us to have better predictions regarding factor allocations. These predictions could be testable, since some data now allows to distinguish screening costs from the compensation of risk in the cost of credit charged by the banking sector to individual firms. This avenue is left open for future research.

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A Additional regression results

Table 8: Interest rate on new loans and firm size, regression results by year

	Dependent variable:											
					Interest rate on new loans			All loans				
	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Medium-sized enterprises	0.609***	0.688***	0.430***	0.215***	0.124***	0.192***	0.442***	0.291***	0.297***	0.328***	0.771***	0.881***
	(0.019)	(0.017)	(0.026)	(0.028)	(0.028)	(0.026)	(0.024)	(0.025)	(0.010)	(0.009)	(0.009)	(0.011)
Small enterprises	1.285***	1.162***	1.816***	0.903***	0.772***	0.650***	0.664***	0.413***	0.464***	0.649***	1.060***	0.977***
	(0.018)	(0.017)	(0.025)	(0.027)	(0.028)	(0.026)	(0.023)	(0.025)	(0.010)	(0.009)	(0.009)	(0.010)
Microenterprises	1.905***	1.506***	3.093***	2.116***	1.530***	1.244***	1.748***	1.546***	1.356***	1.280***	1.629***	1.663***
	(0.021)	(0.019)	(0.029)	(0.030)	(0.029)	(0.028)	(0.024)	(0.027)	(0.013)	(0.012)	(0.012)	(0.015)
Quarter FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm sector FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm rating FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Loan amount FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Loan maturity FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Loan type FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year x Firm rating FE	No	No	No	No	No	No	No	No	No	No	No	No
Year x Loan amount FE	No	No	No	No	No	No	No	No	No	No	No	No
Year x Loan maturity FE	No	No	No	No	No	No	No	No	No	No	No	No
Year x Loan type FE	No	No	No	No	No	No	No	No	No	No	No	No
Observations	190,674	226,196	130,860	150,819	187,371	195,505	190,039	178,282	237,036	234,450	252,882	148,923
Adjusted R^2	0.301	0.238	0.464	0.458	0.316	0.424	0.474	0.451	0.300	0.352	0.413	0.430

*p<0.1; **p<0.05; ***p<0.01

B Bond pricing example

I illustrate how bond prices vary if the process governing entrepreneurial skills follows an AR(1) process of the following form:

$$\ln z_{it+1} = \mu + \rho \ln z_{it} + \sigma \epsilon_{it+1}$$

where ρ is the persistence of the process, σ its standard deviation, μ is adjusted so that the unconditional mean of the productivity process is equal to one, and ϵ_{it+1} is an i.i.d. Gaussian shock.

Note that in this case, I can express the relevant moments of the z distribution, necessary to price the bond issued by an entrepreneur i. Indeed, we have:

$$\mathbb{F}(\underline{z}_{it}|z_{it-1}) = \int_{z=0}^{\underline{z}_{it}} \frac{1}{z\sqrt{2\pi\sigma^2}} e^{\frac{-(\ln(z)-\rho\ln(z_{it-1})-\mu)^2}{2\sigma^2}} dz$$

$$= \frac{1}{2} + \frac{1}{2} erf\left(\frac{\ln(\underline{z}_{it}) - \rho\ln(z_{it-1}) - \mu}{\sqrt{2\sigma^2}}\right)$$

$$\mathbb{E}\left[z|z < \underline{z}_{it}|z_{it-1}\right] = \int_{z=0}^{\underline{z}_{it}} z\frac{1}{z\sqrt{2\pi\sigma^2}} e^{\frac{-(\ln(z)-\rho\ln(z_{it-1})-\mu)^2}{2\sigma^2}} dz$$

$$= e^{\rho\ln(z_{it-1})+\mu+\frac{\sigma^2}{2}}\left[1 - \Phi\left(\frac{\rho\ln(z_{it-1}) + \mu + \sigma^2 - \ln(\underline{z}_{it})}{\sigma}\right)\right]$$

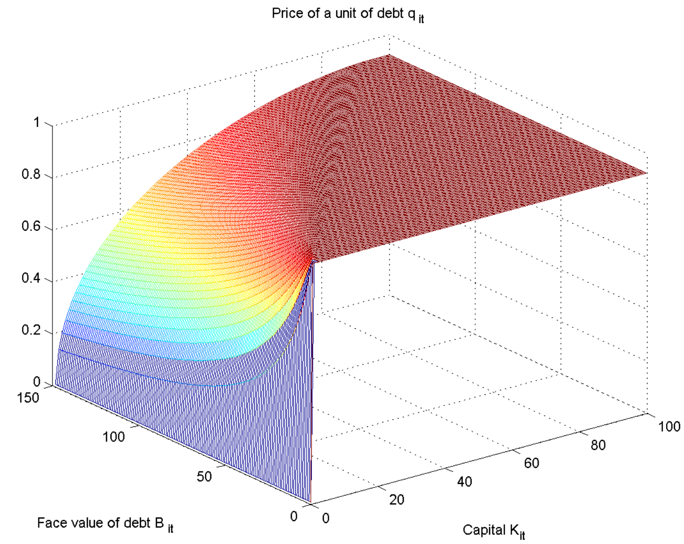
$$= z_{it-1}^{\rho} e^{\mu+\frac{\sigma^2}{2}}\left[1 - \Phi\left(\frac{\ln\left(\frac{z_{it-1}^{\rho}}{\underline{z}_{it}}\right) + \mu}{\sigma} + \sigma\right)\right]$$

$$\mathbb{E}\left[\left(\frac{z}{\underline{z}_{it}}\right)^{\frac{1}{1-(1-\alpha)\eta}} |z < \underline{z}_{it}\right] = \left(\frac{z_{it-1}^{\rho}}{\underline{z}_{it}}\right)^{\frac{1}{1-(1-\alpha)\eta}} e^{\frac{\mu}{1-(1-\alpha)\eta} + \frac{\left(\frac{\sigma}{1-(1-\alpha)\eta}\right)^2}{2}}$$

$$\times\left[1 - \Phi\left(\frac{\ln\left(\frac{z_{it-1}^{\rho}}{\underline{z}_{it}}\right) + \mu}{\sigma} + \frac{\sigma}{1-(1-\alpha)\eta}\right)\right]$$

Figure 10 plots the price of the bond, as a function of the capital and debt choices, for some

Figure 10: Bond price - Example



parameter values and an initial productivity level z_{it-1}. Note that, for a given level of capital, debt is risk free until it reaches the value corresponding to that of undepreciated capital, net of wage payments to the entrepreneur. After that, debt becomes risky, since the corresponding collateral consists of profits, which depend on the realization of the productivity shock. Hence, the price of the bond falls. Figure 11 illustrates the same phenomenon, but plots the interest rate (defined as the inverse of the price of the bond, minus one, and expressed in percentages).

More interestingly, Figure 12 shows the impact of the initial productivity level, for a fixed level of capital. In this plot, we can see that, for a given level of desired capital and for a given face value of debt, the price of this debt is higher for a firm characterized by a lower initial level of productivity. This is due to the fact that, since productivity is persistence, the probability that the less productive firm faces a realization of the productivity shock below the default cutoff value is higher. This drives the price of the bond down and the interest rate up.

Figure 13 offers another exercise of comparative statics. It examines the impact of a higher cost of bankruptcy (or lower χ on the price. Evidently, higher bankruptcy costs imply that the payoff of the bond is lower in case of default. This drives the bond price down and the interest rate up, for any given choice of capital and debt, and any initial productivity value.

Figure 11: Interest rate - Example

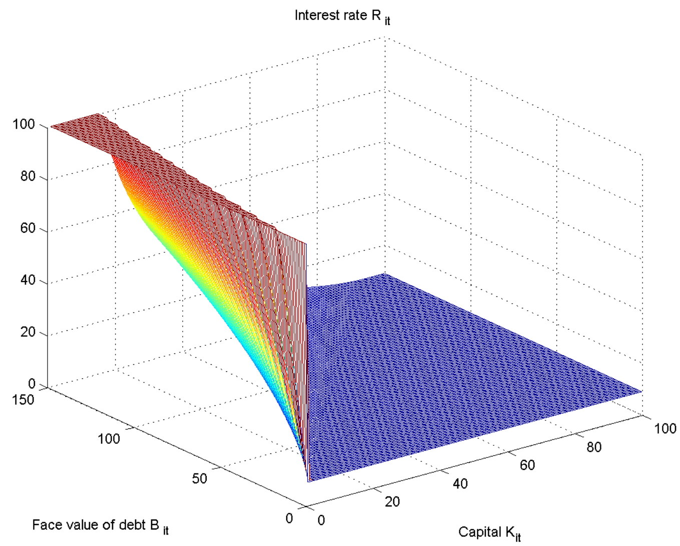


Figure 12: Impact of the initial productivity level on the interest rate - Example

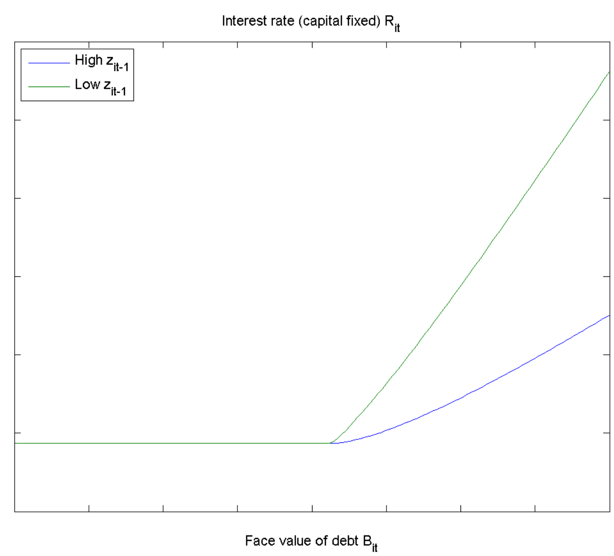


Figure 13: Impact of the bankruptcy costs on the interest rate - Example